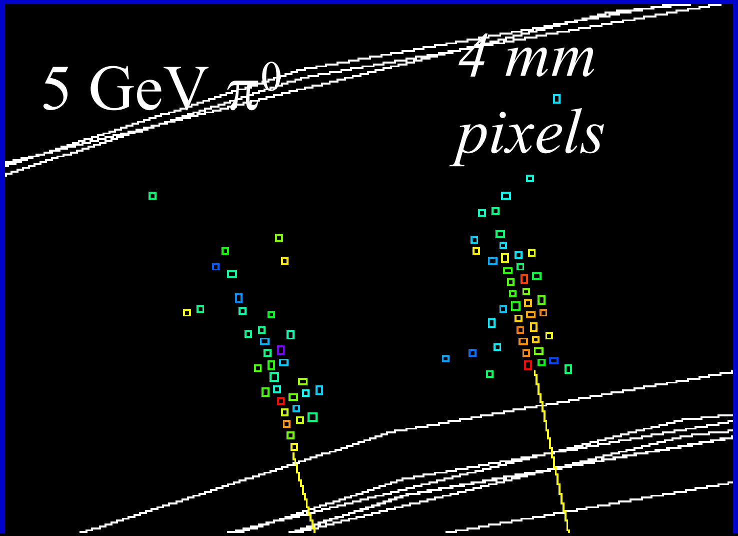
Towards Jet Specific Energy Resolution: Investigating π^0 Kinematic Fits

EM calorimeters under consideration for ILC have unprecedented potential for photon position resolution.

Can this be used to measure π^0 energies very well and by extension hadronic jets ?

Also see talks 2005-2007 on π^0 KF basics and initial forays into applying to hadronic events.

(latest: ALCPG07 for more details.)

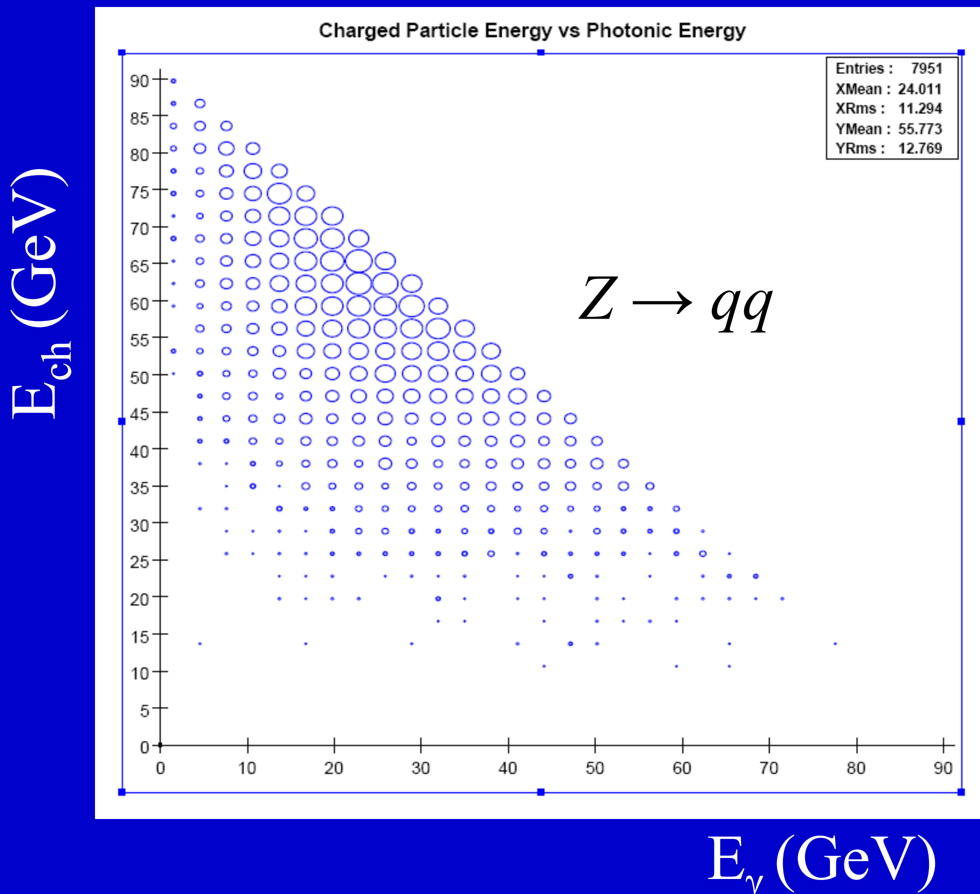


1. Motivation: Jet Specific Energy Resolution & Physics
2. π^0 kinematic fitting
3. Improvements in π^0 energy resolution
4. Applying to hadronic jets

Graham W. Wilson and Brian van Doren

Advanced Particle Flow

$$E_{jet} = E_{ch} + E_{\gamma} + E_{NH}$$



How does PFA depend on (f_{ch}, f_{γ}) *?*

On (n_{ch}, n_{γ})*? etc.*

Develop ***jet specific energy resolution*** formalism.

Take advantage of knowledge of jet energy errors jet per jet.

Non-Gaussian resolution function is not a cardinal sin – it is a potentially exploitable feature.

Will eventually need detailed understanding at individual event level inside PF algorithms.

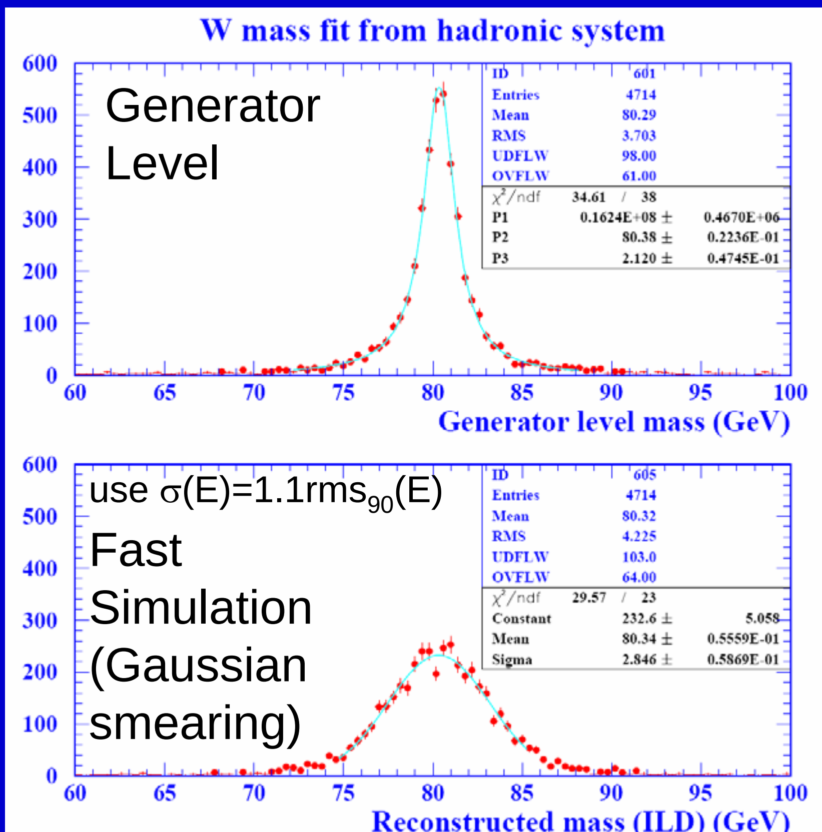
As a first step, take advantage of error knowledge on the fitted photon component (under the π^0 mass hypothesis).

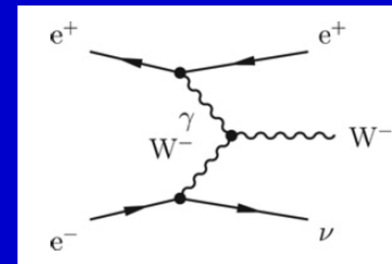
May be most useful in the near-term in the “no-confusion” limit.

Example (New) Physics Analysis

Possible 1 TeV benchmark ?

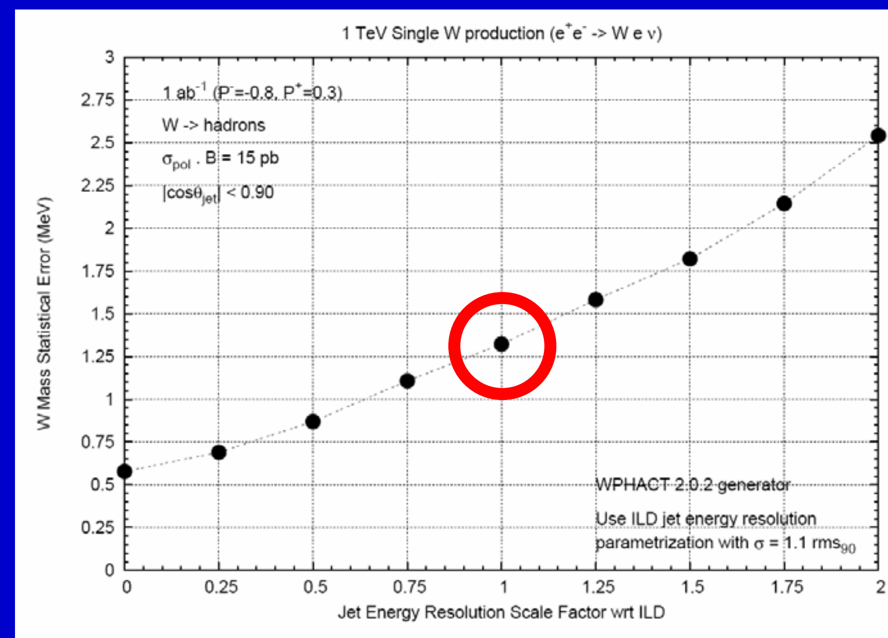
Single W study at $\sqrt{s}$ = 1TeV





W → q q
(jets are not so energetic)

Is this useful for physics ? Example m_W.



=> Further E_{jet} resolution improvement and knowledge very desirable

Potentially very useful ! (Especially, if the really challenging requirements on jet energy scale and calibration can be met !)

Absolute Jet Energy Scale

- One self-contained approach for PFA could be bottom-up using known particle masses.
  - Momentum scale (J/ψ)
  - Photon scale (π^0)
  - K^0_L scale (ϕ)
  - n scale (Σ)
  - nbar scale (Σ)
- Probably unrealistic as the only method.
  - But may point to the need for substantial statistics at the Z.

π^0 Kinematic Fitting

π^0's and Particle Flow

- Particle Flow
  - Charged particles => TRACKER => 62%
  - **Photons => ECAL => 28%**
  - Neutral hadrons => HCAL => 10%
- Photons
  - Prompt Photons (can assume vtx = (0,0,0))
    - **π^0 (About 95% of the photon energy content at the Z)**
    - η, η' etc.
    - Lone photons (eg. $\omega \rightarrow \pi^0 \gamma$)
  - Non-prompt Photons
    - $K^0_S \rightarrow \pi^0 \pi^0$
    - $\Lambda \rightarrow \pi^0 n$
- So, as you know, most photons do come from prompt π^0's, we do know the π^0 mass, and they interact in well understood ways !
- So, for correctly paired photons, π^0 mass constraint is reasonable, and we have shown that the improvement in estimating $E_{\pi 0}$ can be sizeable.

Detector Resolution

- Both ILD and SiD envisage compact EM calorimeters capable of very precise angular measurements readout every X0 or so.
- Examples:
- Si-W
  - (13 mm^2 cells at R=1.27 m (SiD)
  - (25 mm^2 cells at R=1.85 m (ILD)
  - (50 μm x 50 μm pixels – MAPS option)
- Can identify the photon conversion point in the ECAL with resolution typical of the pixel size largely independent of the photon energy.
- Resolutions in the 0.5 mrad range per projection for 1 GeV photons is at hand (assuming photon is prompt).

Documentation

Working on a paper documenting and extending the foundations of earlier studies. Emphasis is on a generic detector for a wide range of resolution assumptions. Mainly treating the single π^0 case using smeared Monte Carlo.

Applying mass-constrained fits to the energy reconstruction of di-photon resonances with high granularity calorimeters

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E-mail: gwwilson@ku.edu

ABSTRACT: Mass-constrained fits to correctly matched pairs of photons are investigated and the improvements in di-photon energy resolution are quantified for the ubiquitous π^0 for a range of π^0 energies, center-of-mass decay angles, and assumptions on photon energy and angular resolution.

π^0 Kinematic Fitting I

- For simplicity, (old 3-variable studies) used the following measured experimental quantities:

  E_1 (Energy of photon 1)

  E_2 (Energy of photon 2)

  ψ_{12} (3-d opening angle of photons 1 and 2)

- Fit using
  - 3 variables, $\mathbf{x} = (E_1, \; E_2, \; 2(1 - \cos\psi_{12}))$
  - a diagonal error matrix

    (assumes individual γ's are completely resolved and measured independently)
  - and the constraint equation

    $m_{\pi^0}^2 = 2 E_1 E_2 (1 - \cos\psi_{12}) = x_1 x_2 x_3$

π^0 Kinematic Fitting II

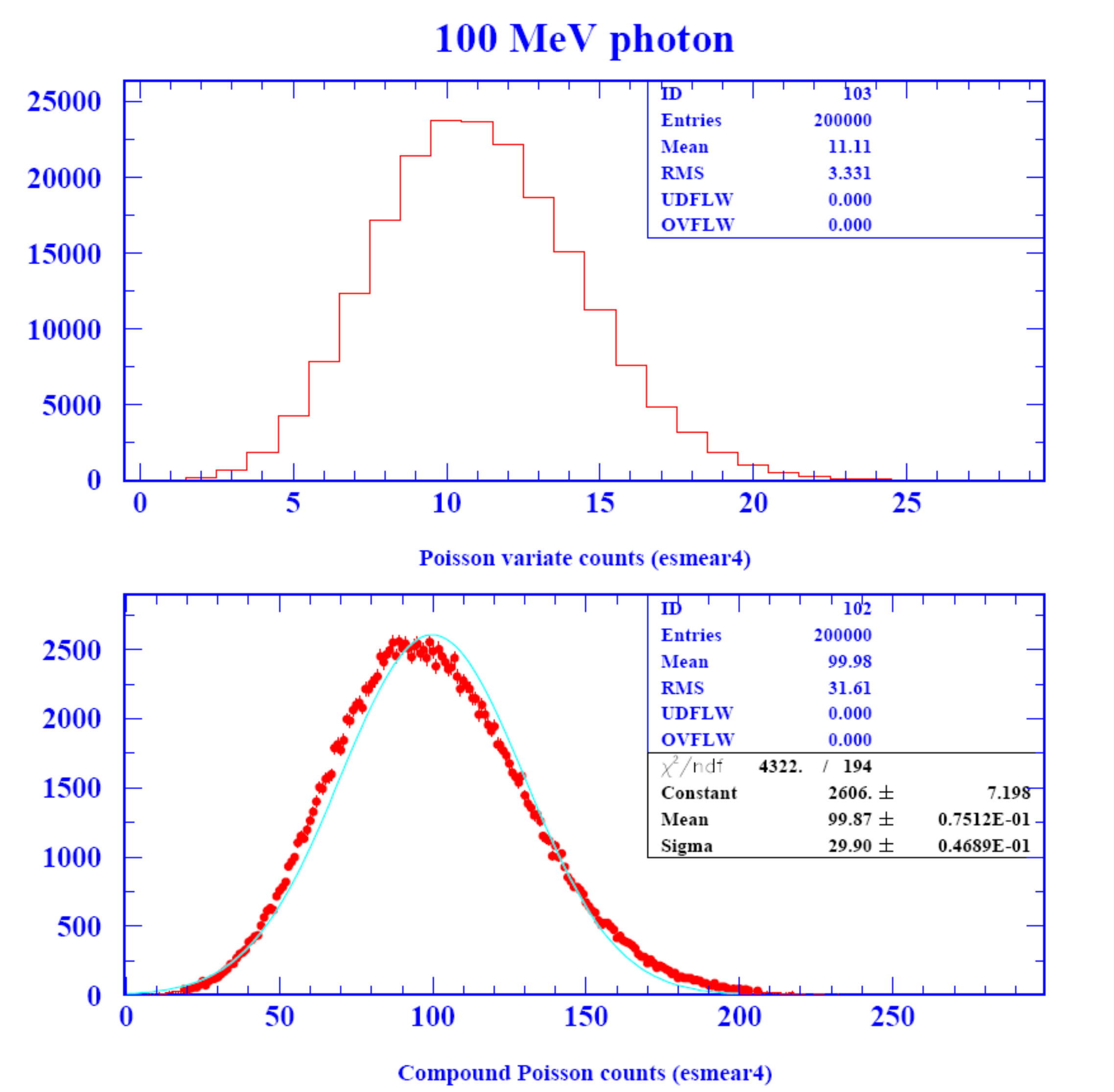
- The new 6-variable study uses (E, θ, φ) for each photon.
- Still a diagonal error matrix.
- Implementations:
  - 3 variable: analytic
  - 3 variable: Blobel F77 fitter
  - 6 variable: Blobel F77 fitter
  - 6 variable: MarlinKinFit (Brian)

Have been able to cross-check all four with identical inputs.

- 6-variable advantages:
  - More realistic angular resolution implementation
  - Assess improvements in π^0 direction

Energy Smearing and Detection Threshold

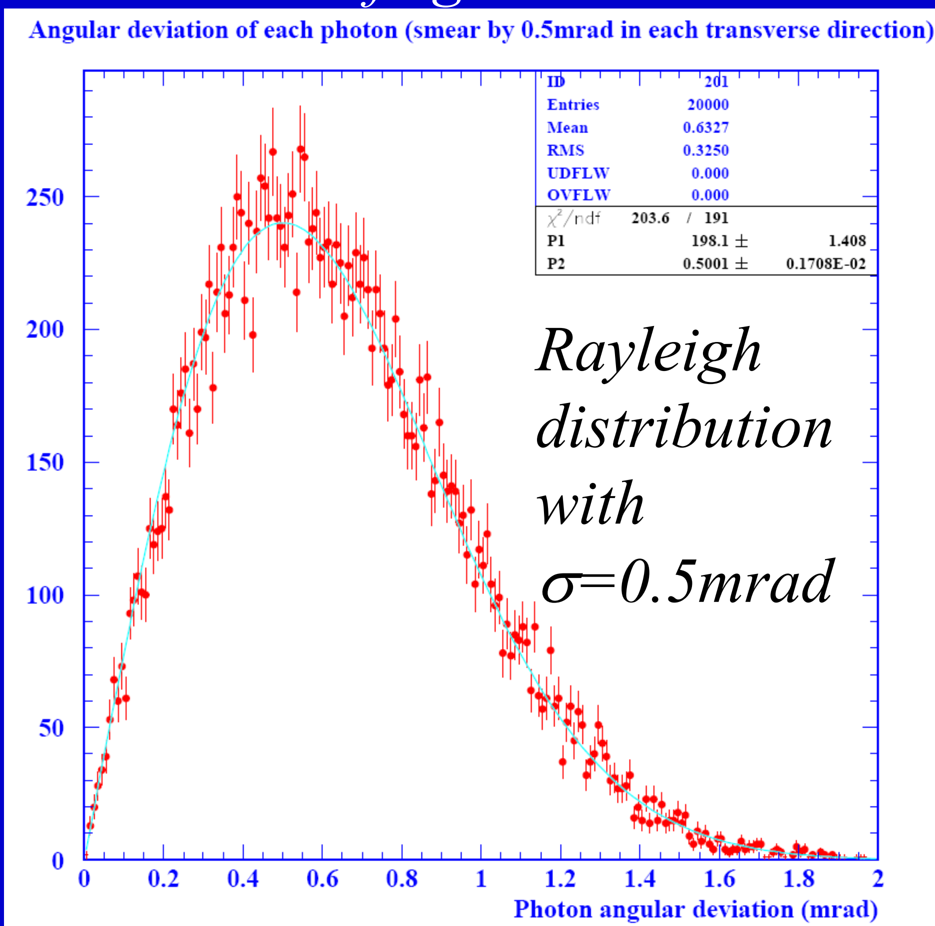
- Previously had used Gaussian energy smearing.
  - $\sigma_E/E = \alpha/\sqrt{E}$
  - Non-negligible probability of –ve energy.
- Elected to smear the photon energies using a Compound Poisson distribution (reasonably physically motivated as a model of branching processes).
- Impose a minimum detection threshold at E (GeV) > 2 α^2
- For α=0.16, Emin = 0.05 GeV

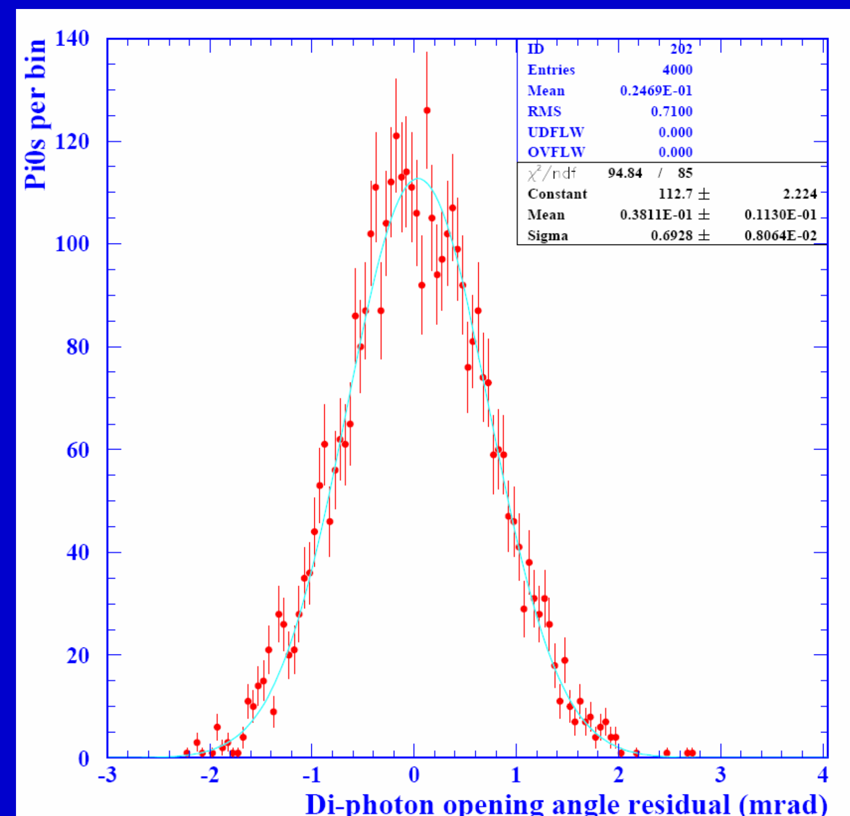


Smearing the Photon Angular Resolution

Photons are assumed to be prompt. So angular resolution is equivalent to position resolution in the ECAL for this application

Photons are smeared independently in "x" and "y" by Gaussians with width of eg. σ = 0.5mrad independent of energy





$Err(\psi_{12}) = \sqrt{2}\,\sigma$ *(previous thinking: $Err(\psi_{12}) = 2\,\sigma$!)*

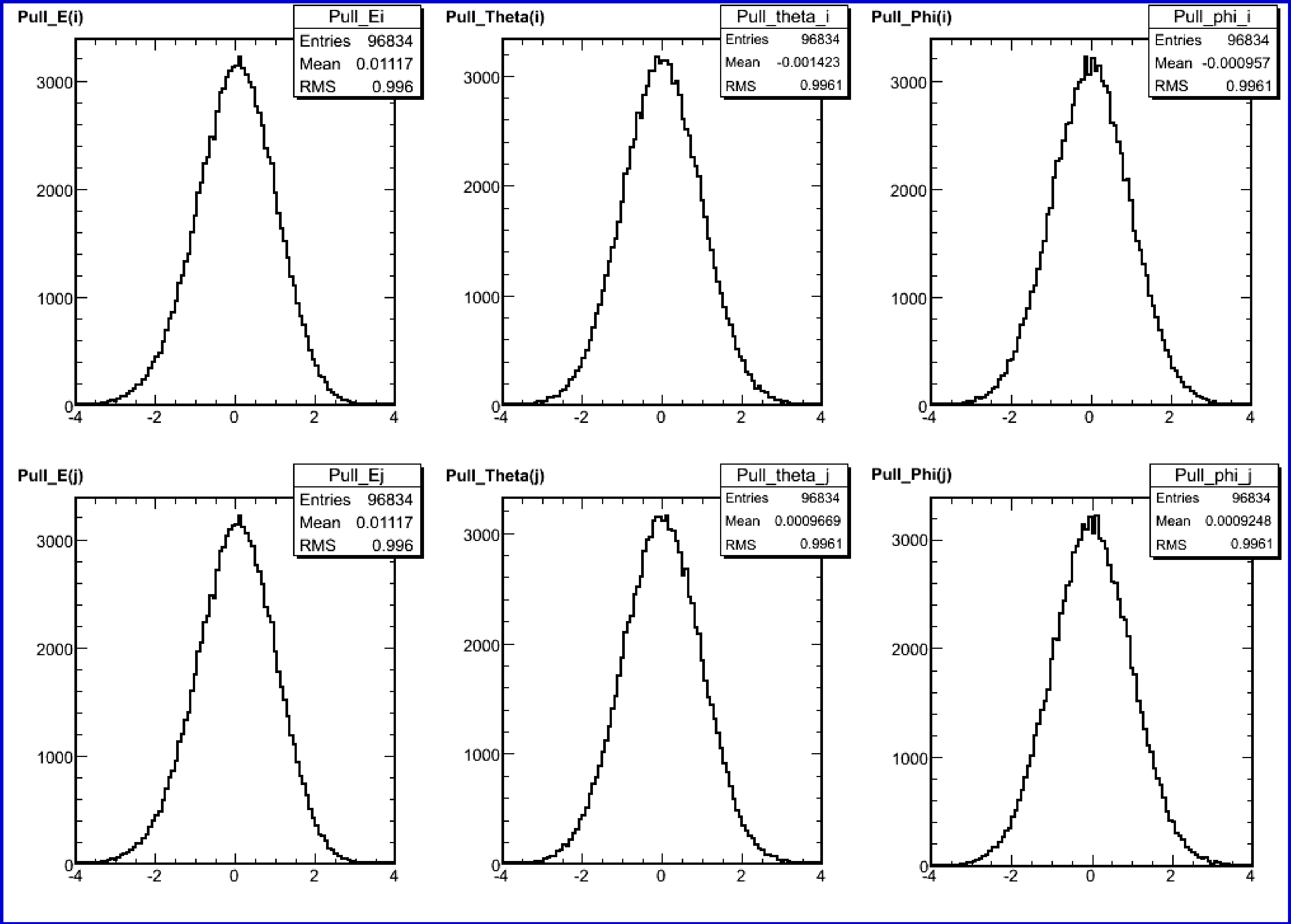
Example Fit

4 GeV π^0, 16%/$\sqrt{E}$, 0.5mr (default assumptions unless stated otherwise)

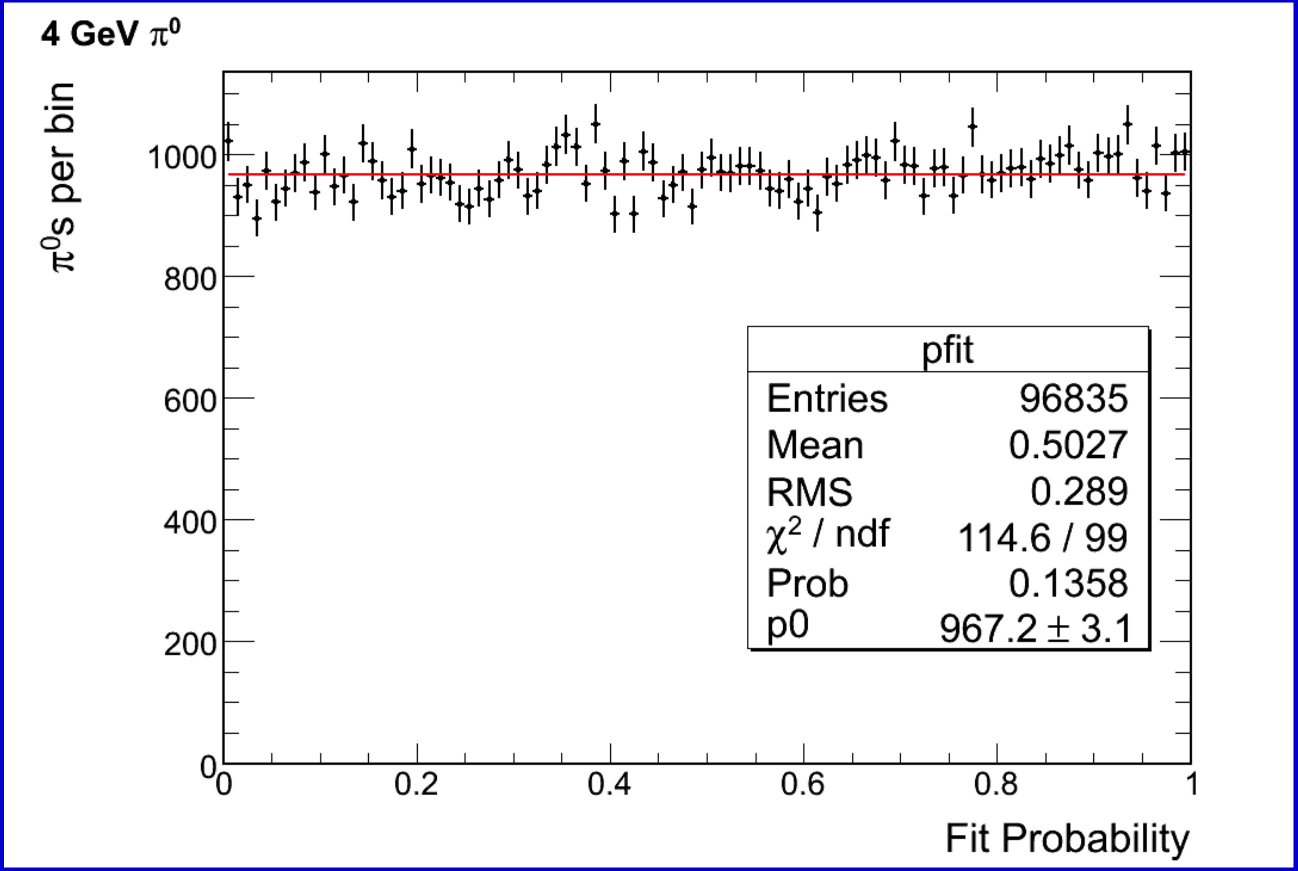
Variable	Measured	3-variable fit	6-variable fit	Pull
E_1	2.468 ± 0.253	2.385 ± 0.192	2.385 ± 0.192	-0.504
E_2	1.679 ± 0.196	1.605 ± 0.130	1.605 ± 0.130	-0.504
$2(1 - \cos\psi_{12})$	$(4.765 \pm 0.0985) \times 10^{-3}$	$(4.759 \pm 0.0977) \times 10^{-3}$		-0.504
θ_1 (mrad)	1608.36 ± 0.50		1608.37 ± 0.50	0.504
θ_2 (mrad)	1619.11 ± 0.50		1619.10 ± 0.50	-0.504
ϕ_1 (mrad)	2196.86 ± 0.50		2196.84 ± 0.50	-0.504
ϕ_2 (mrad)	2128.60 ± 0.50		2128.62 ± 0.50	0.504
m_{π^0} (MeV)	140.5			
$\rho_{E_1E_2}$		-0.9683	-0.9683	
E_{π^0}	4.147 ± 0.320	3.990 ± 0.074	3.990 ± 0.074	
χ^2/ν		0.2543/1		
p_{fit} (%)		61.4		

(Note: the 3 and 6-variable fits are equivalent in terms of energy variables)

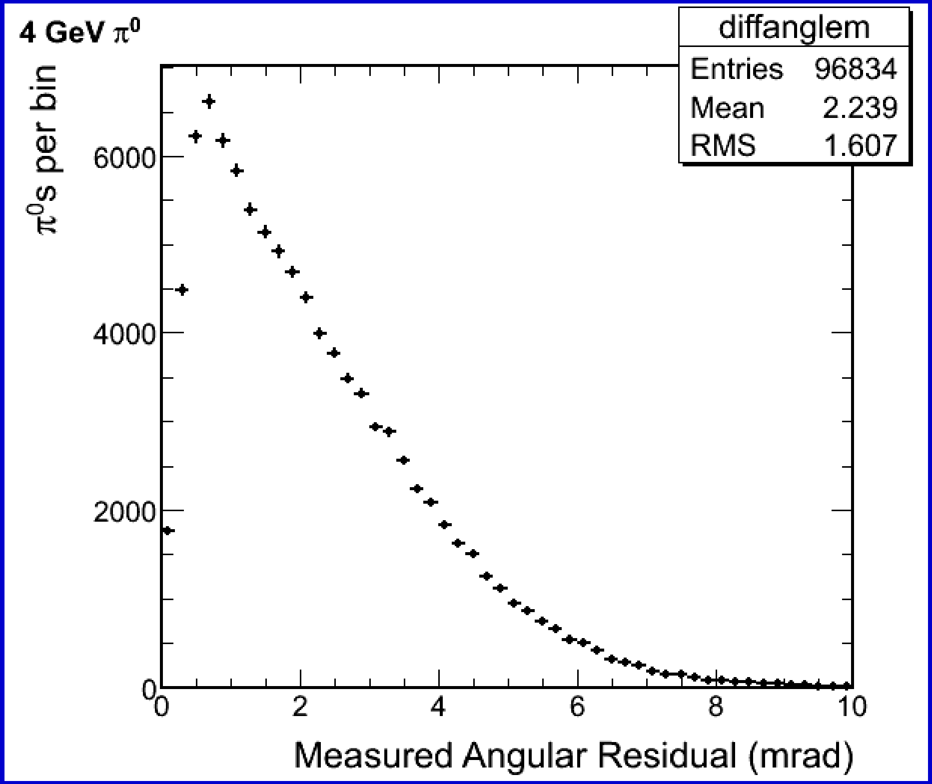
Pull Distributions

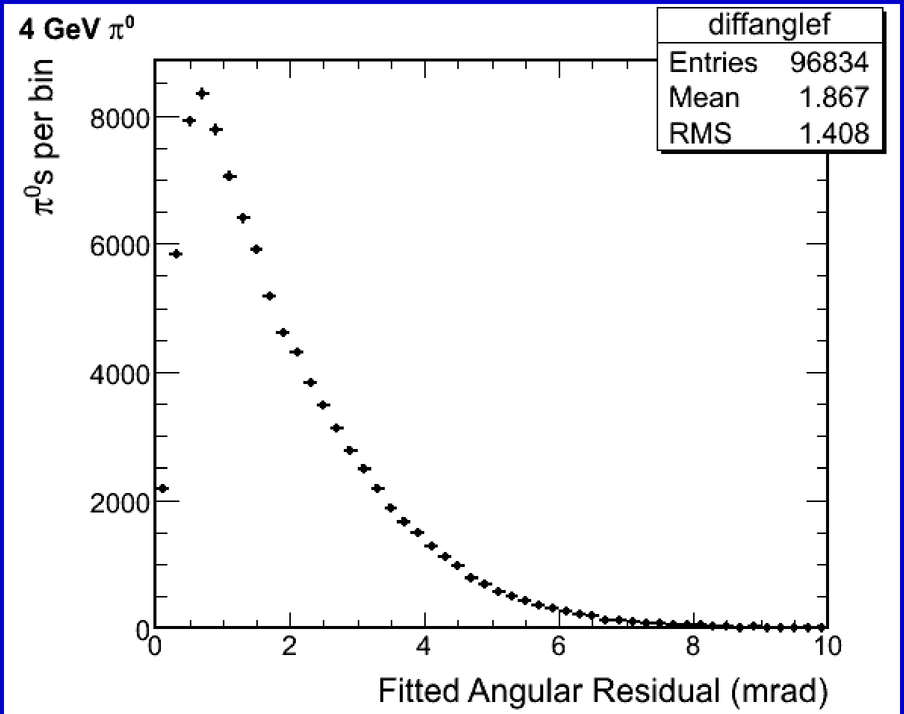


Fit Probability



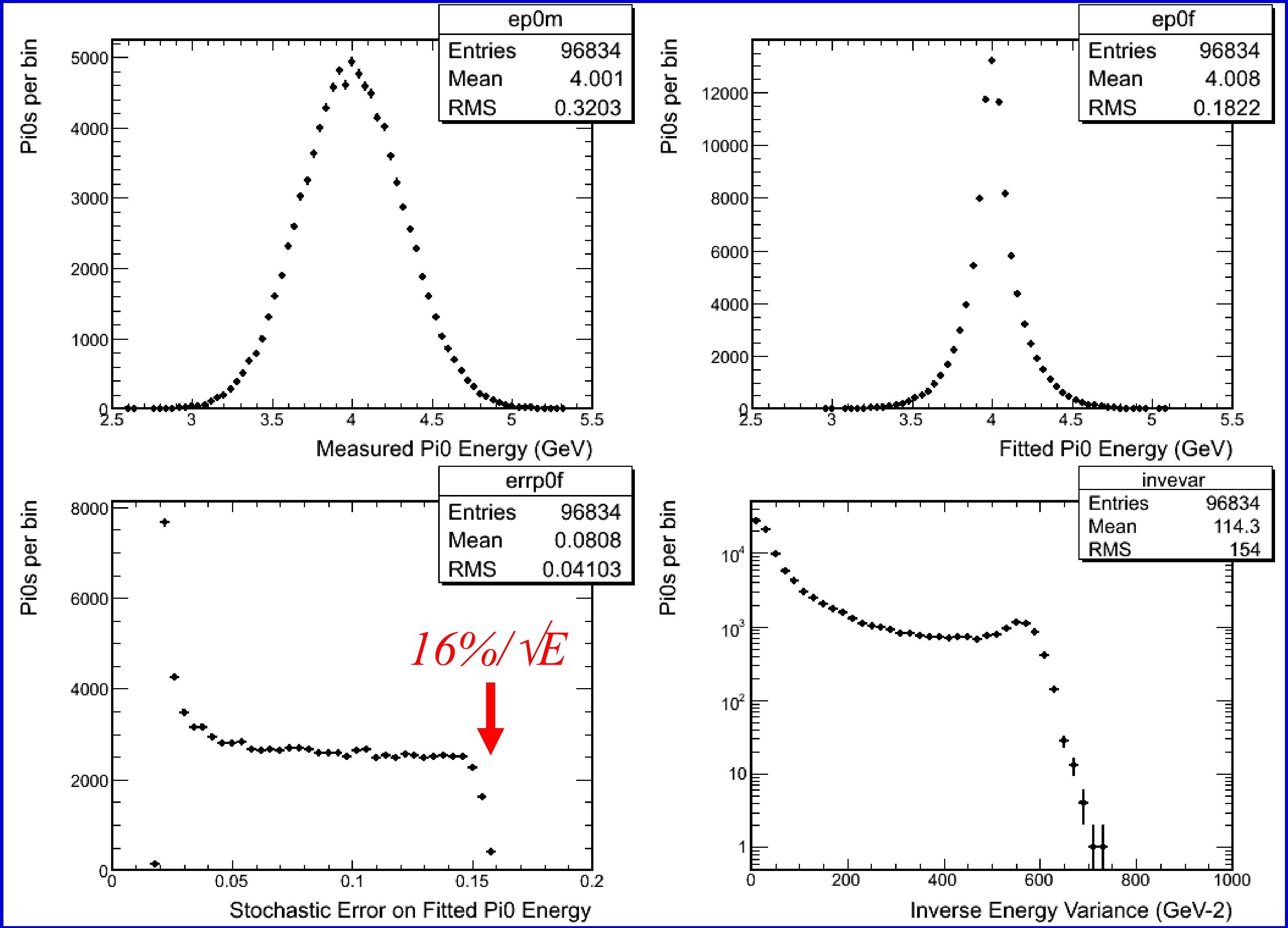
π^0 Angle Improvements





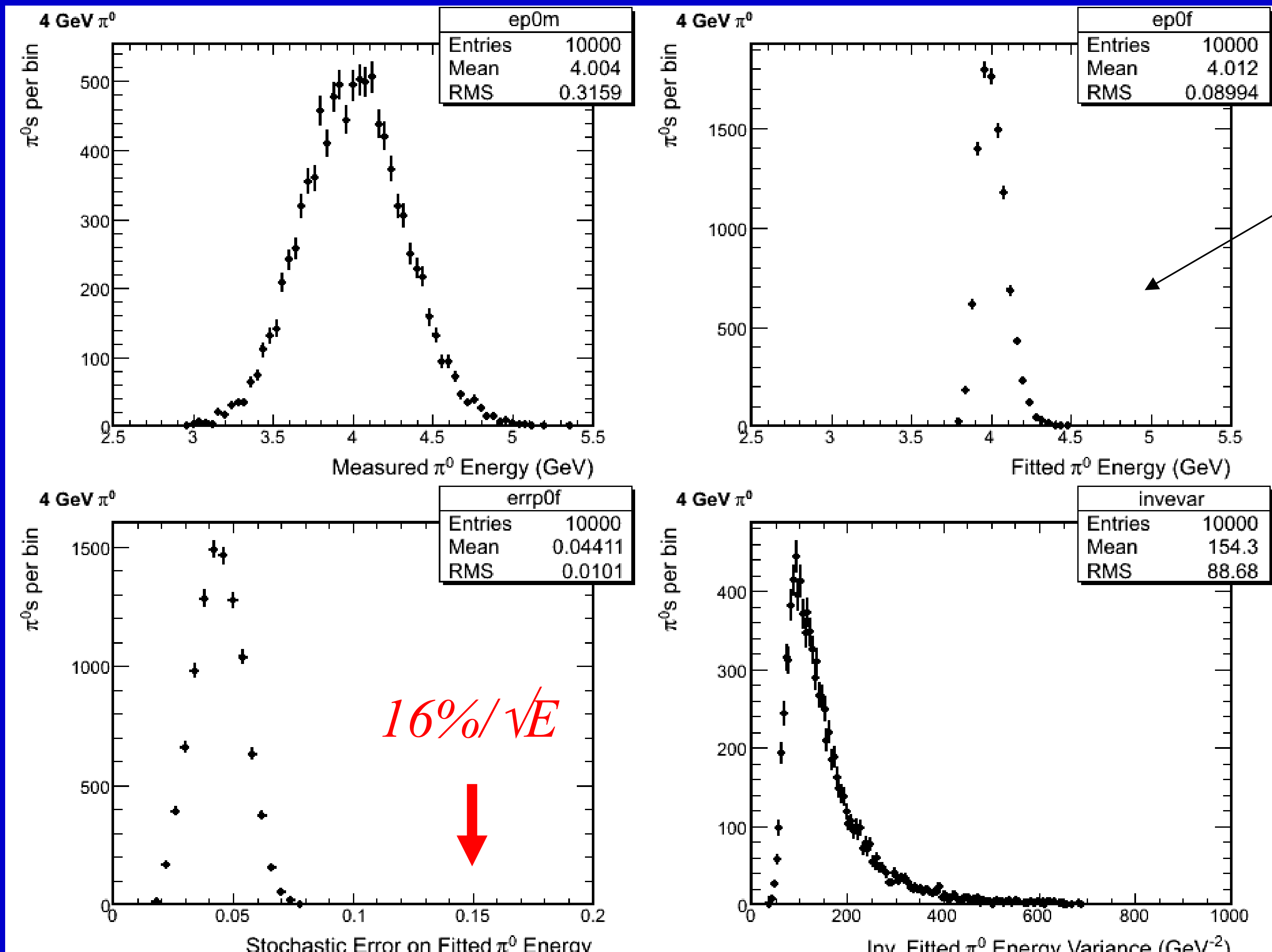
Modest improvements at this energy, but note that this feeds through combinatorically with all other particle pairs in hadronic mass estimates.

4 GeV π^0



ep0m
Entries 96834
Mean 4.001
RMS 0.3203
Pi0s per bin
Measured Pi0 Energy (GeV)
ep0f
Entries 96834
Mean 4.008
RMS 0.1822
Pi0s per bin
Fitted Pi0 Energy (GeV)
errp0f
Entries 96834
Mean 0.0808
RMS 0.04103
Pi0s per bin
16%/√E
Stochastic Error on Fitted Pi0 Energy
invevar
Entries 96834
Mean 114.3
RMS 154
Pi0s per bin
Inverse Energy Variance (GeV-2)

4 GeV π^0 ($\cos\theta^* = 0.25$)

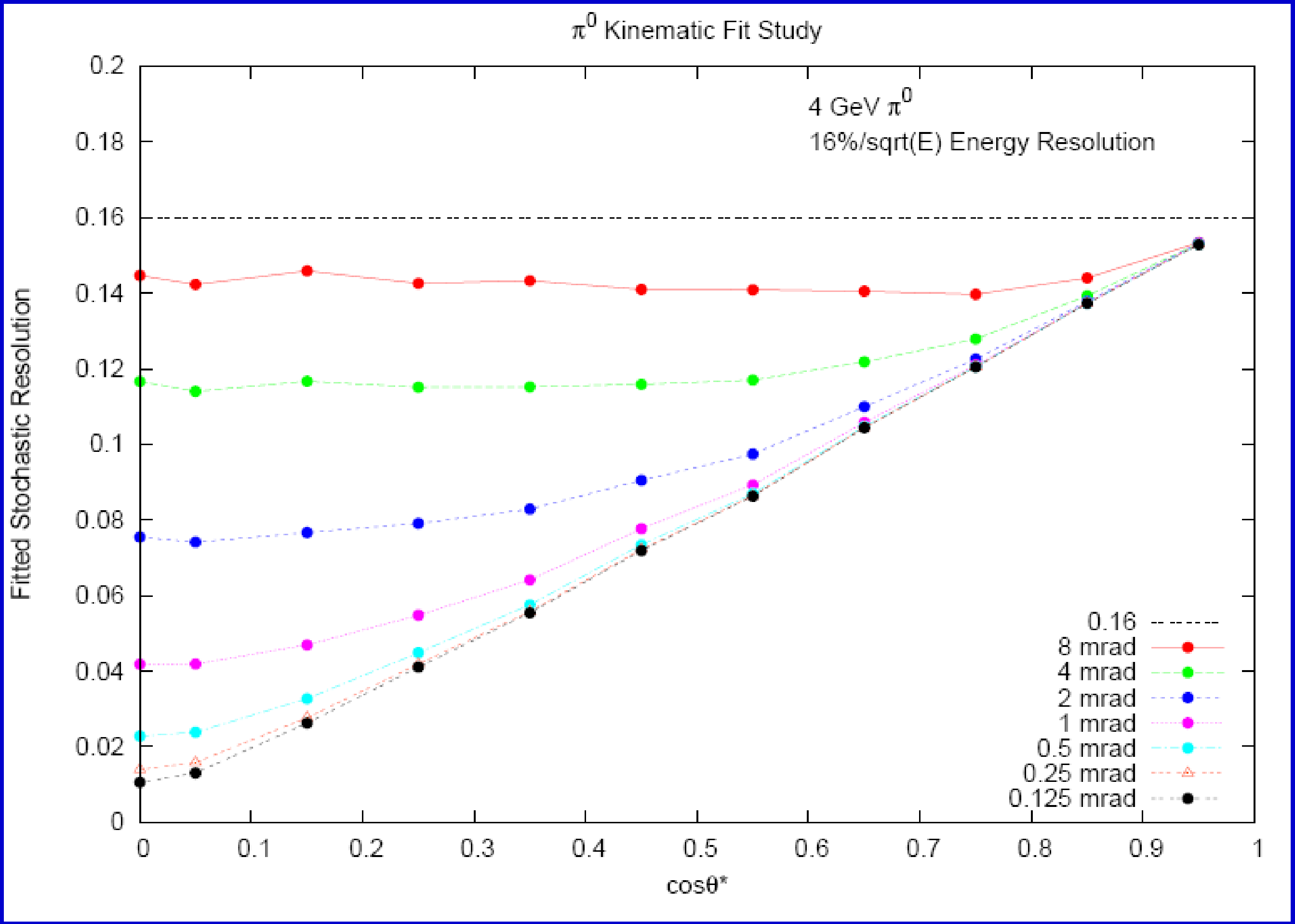


Use mean and RMS of this distribution in following plots for fixed values of $\cos\theta^*$

Fitted π^0 Energy Resolution

Use rms of fitted π^0 energy distribution.

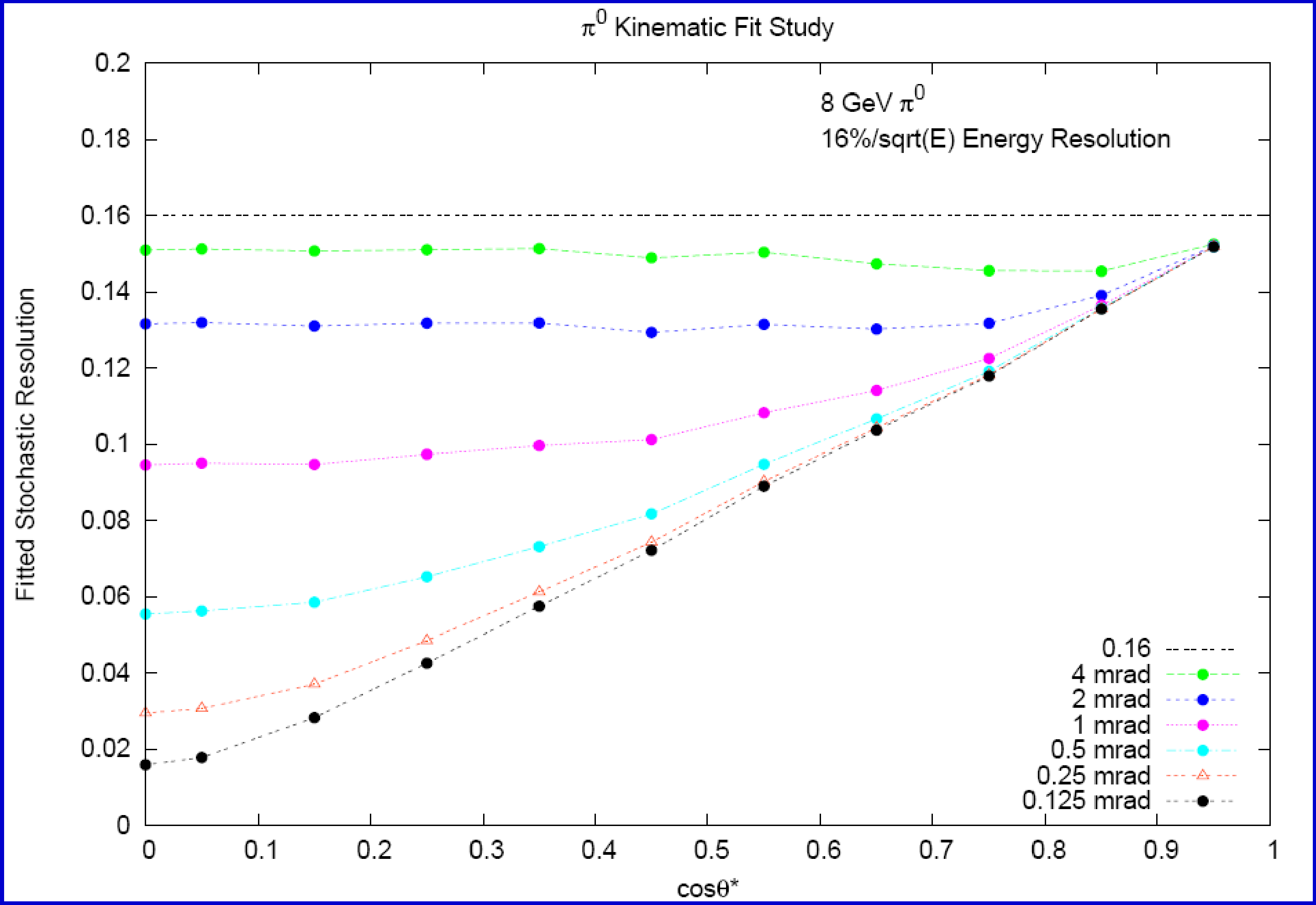
π^0s are generated at fixed $\cos\theta^*$ values



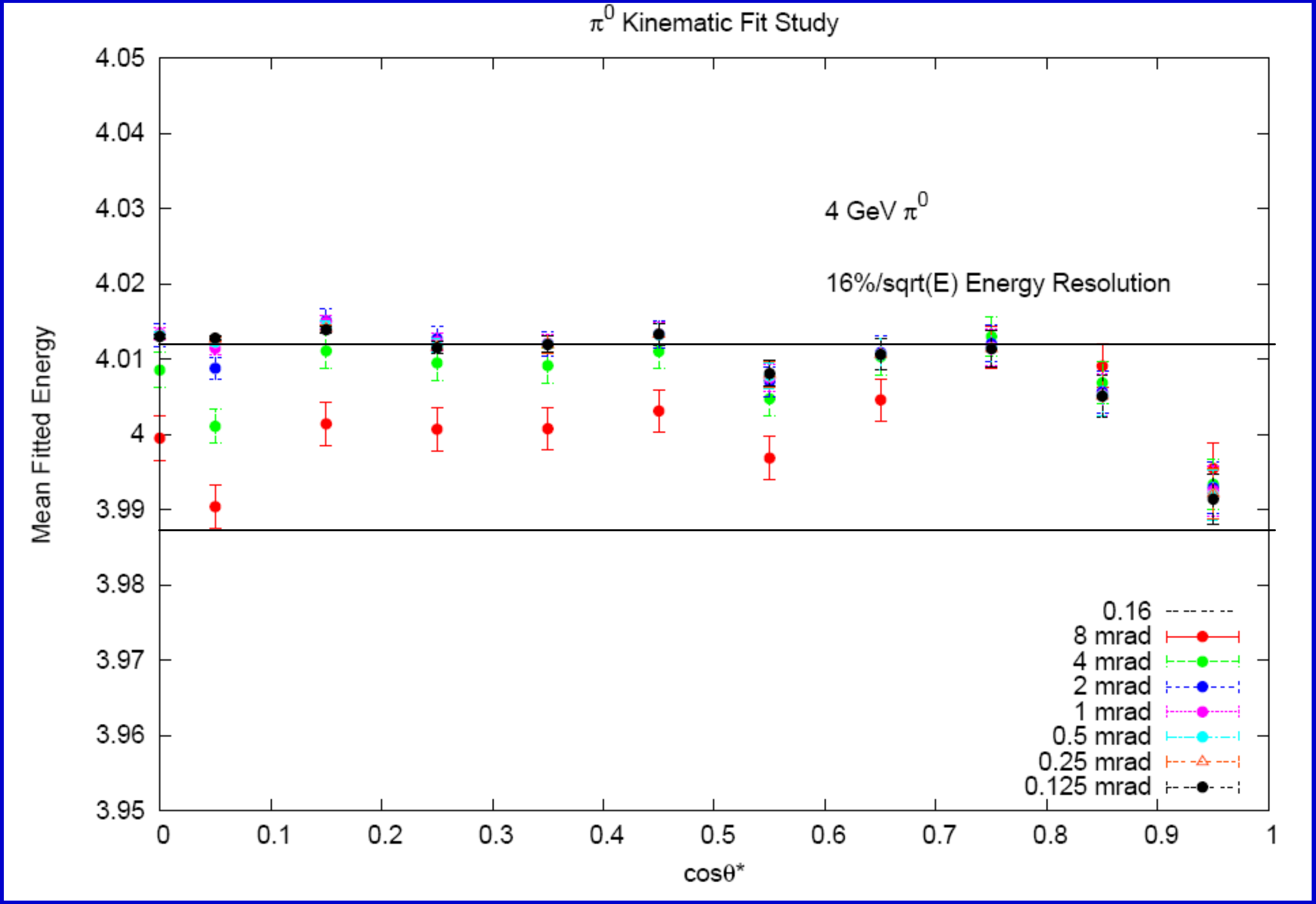
Fitted π^0 Energy Resolution

Use rms of fitted π^0 energy distribution.

π^0s are generated at fixed cosθ values*



Fitted π^0 Energy Bias

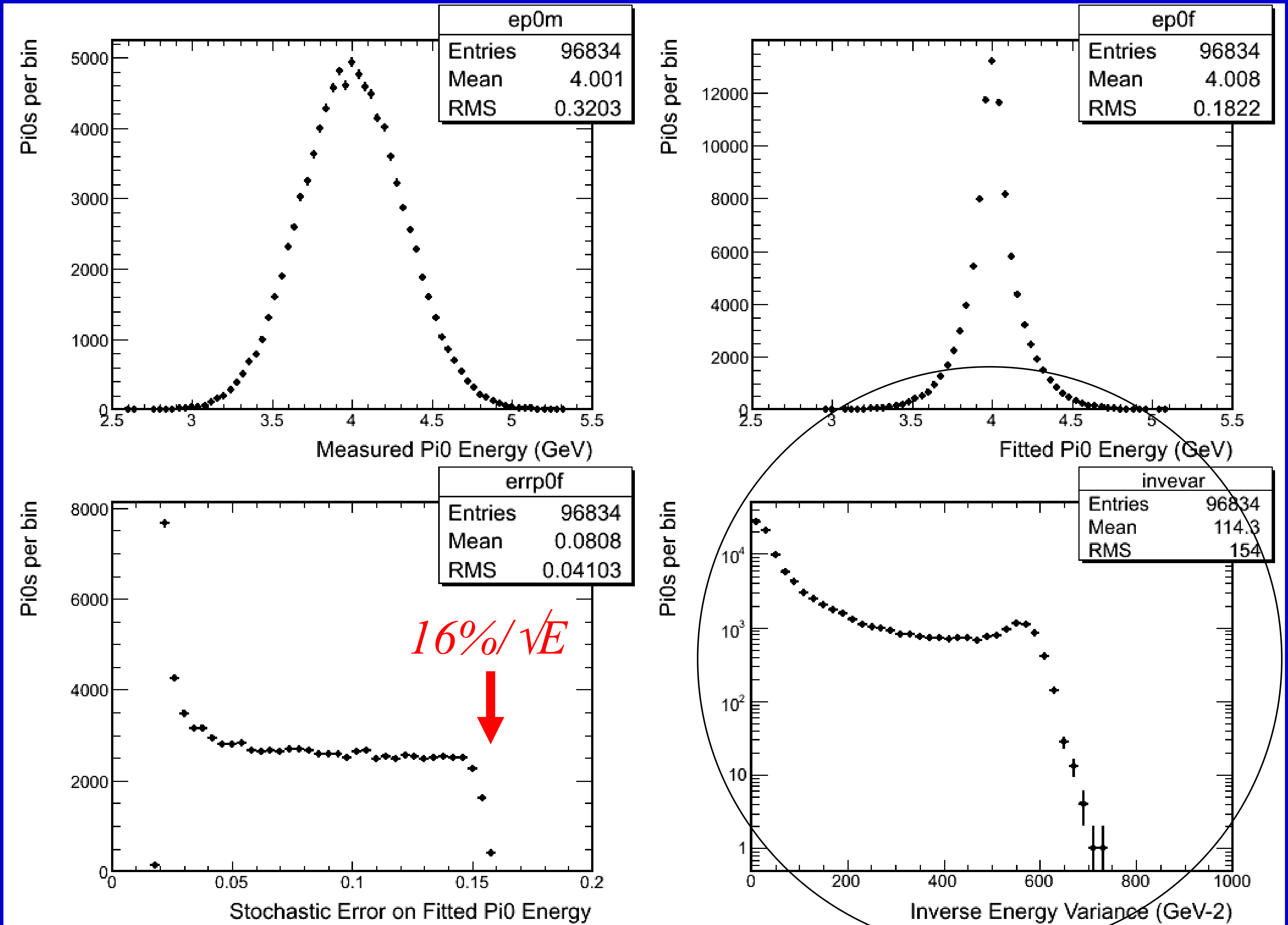


Bias < 0.3%

Weighted Mean

- We can also try and use the π^0 specific energy resolution.
- As an exercise, look at weighting by the fitted energy error of each π^0 in a mono-energetic sample with the usual weight factor of σ_i^{-2}
- In this case, we can define an effective resolution per π^0, $\sigma_* \equiv \sqrt{(1/< \sigma_i^{-2}>)}$, (and also scale this stochastically too).

4 GeV π^0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invevar
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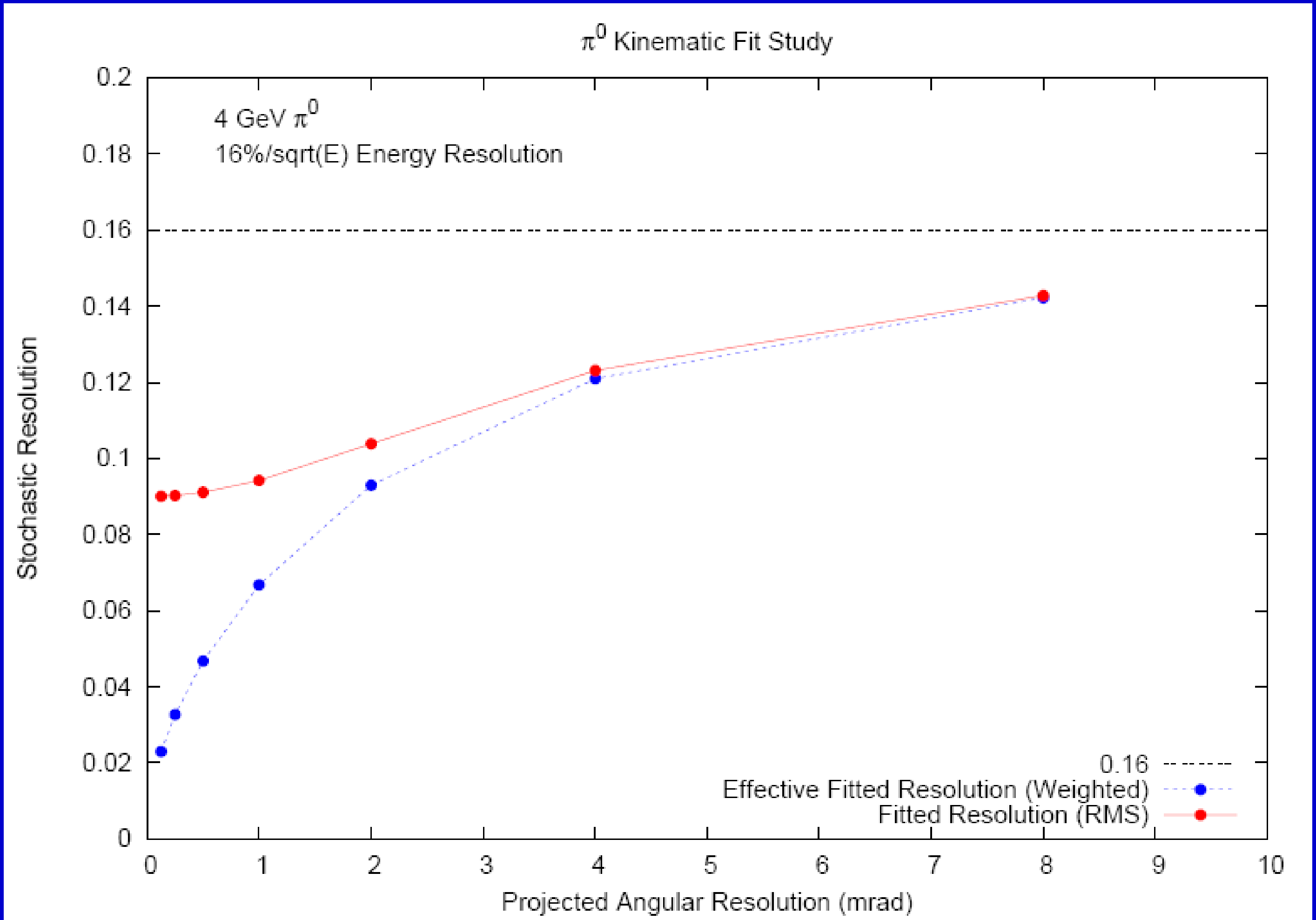
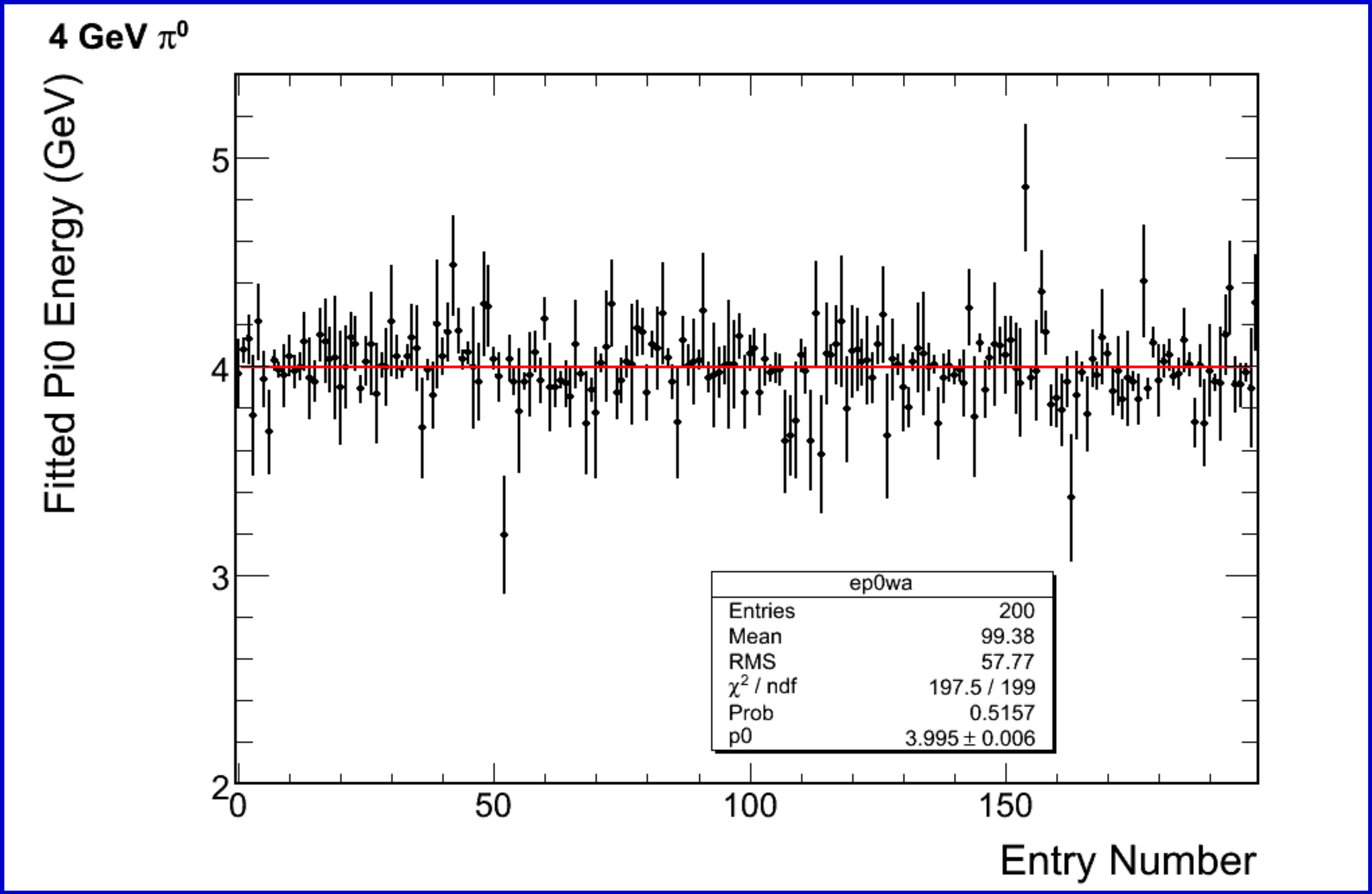
Mean 114.3
RMS 154
Pi0s per bin
Inverse Energy Variance (GeV-2)

Averaging over all cosθ*



Quite an improvement on the apparent statistical error on this "observable"

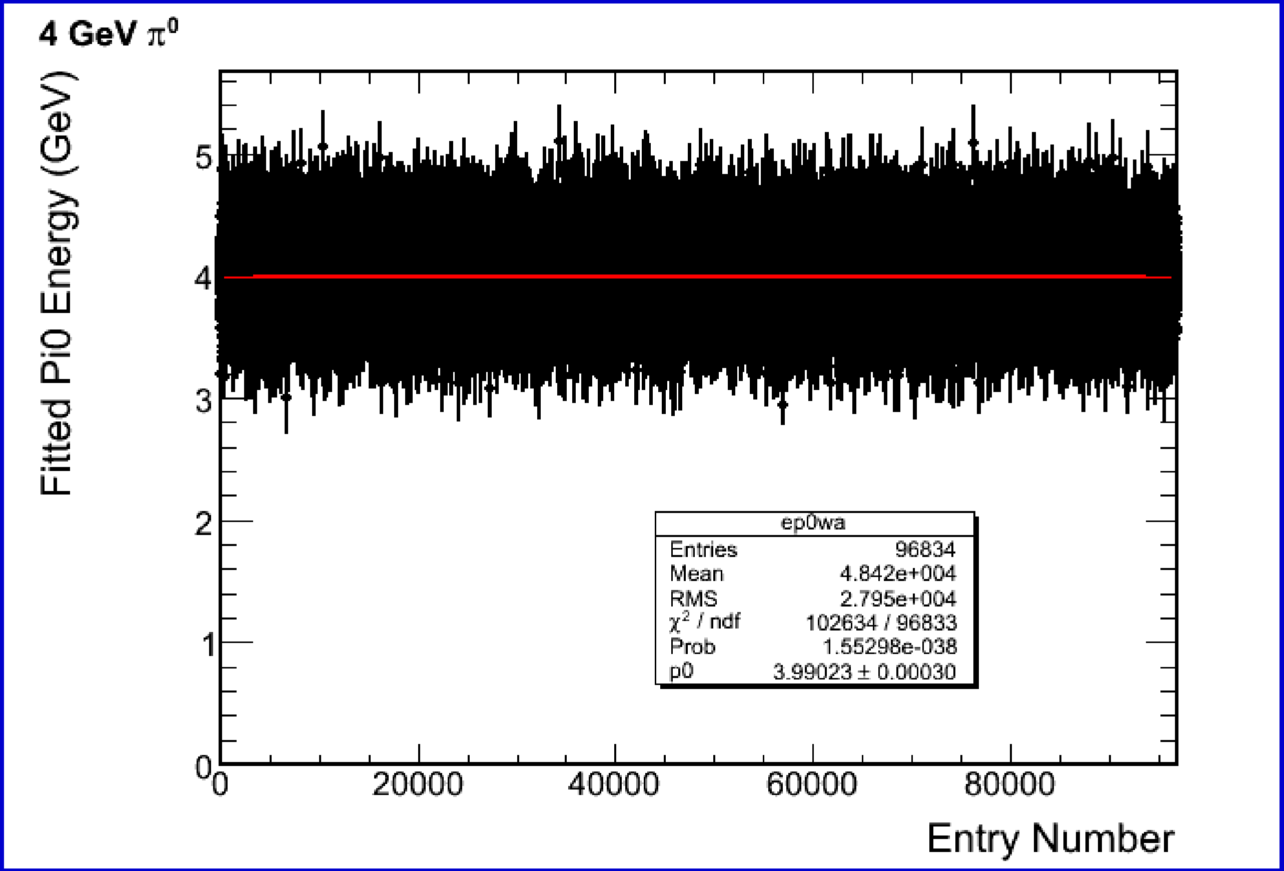
π^0 specific energy resolution



Use fitted error on each π^0 to form weighted average for an ensemble of mono-energetic π^0s.

π^0 specific energy resolution

Large ensemble



*Chi**2/dof small, but not acceptable.*

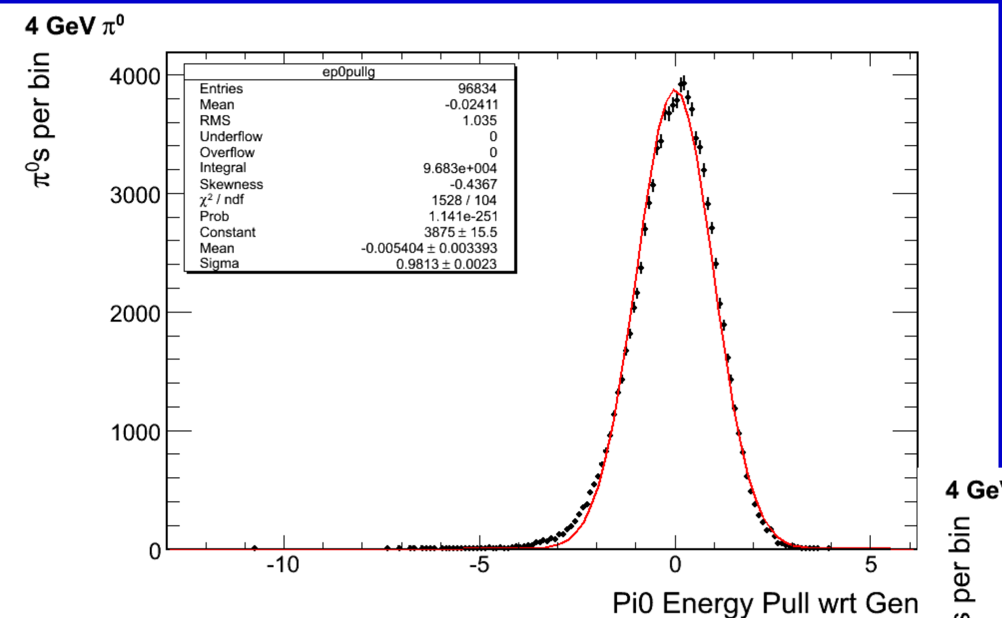
Why ?

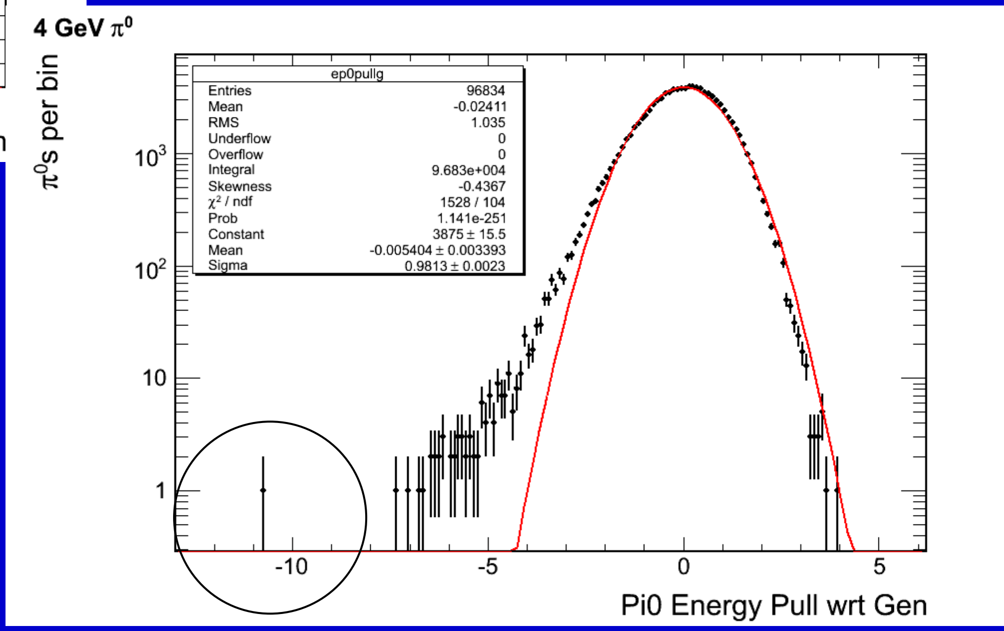
Weighted mean has a bias of around 0.25%

π^0 fit pathology

The fit always adjusts the energies of both photons upwards or downwards according to the measured mass deviation from $m(\pi^0)$.

Sometimes this can lead to a "wrong" fit with small errors





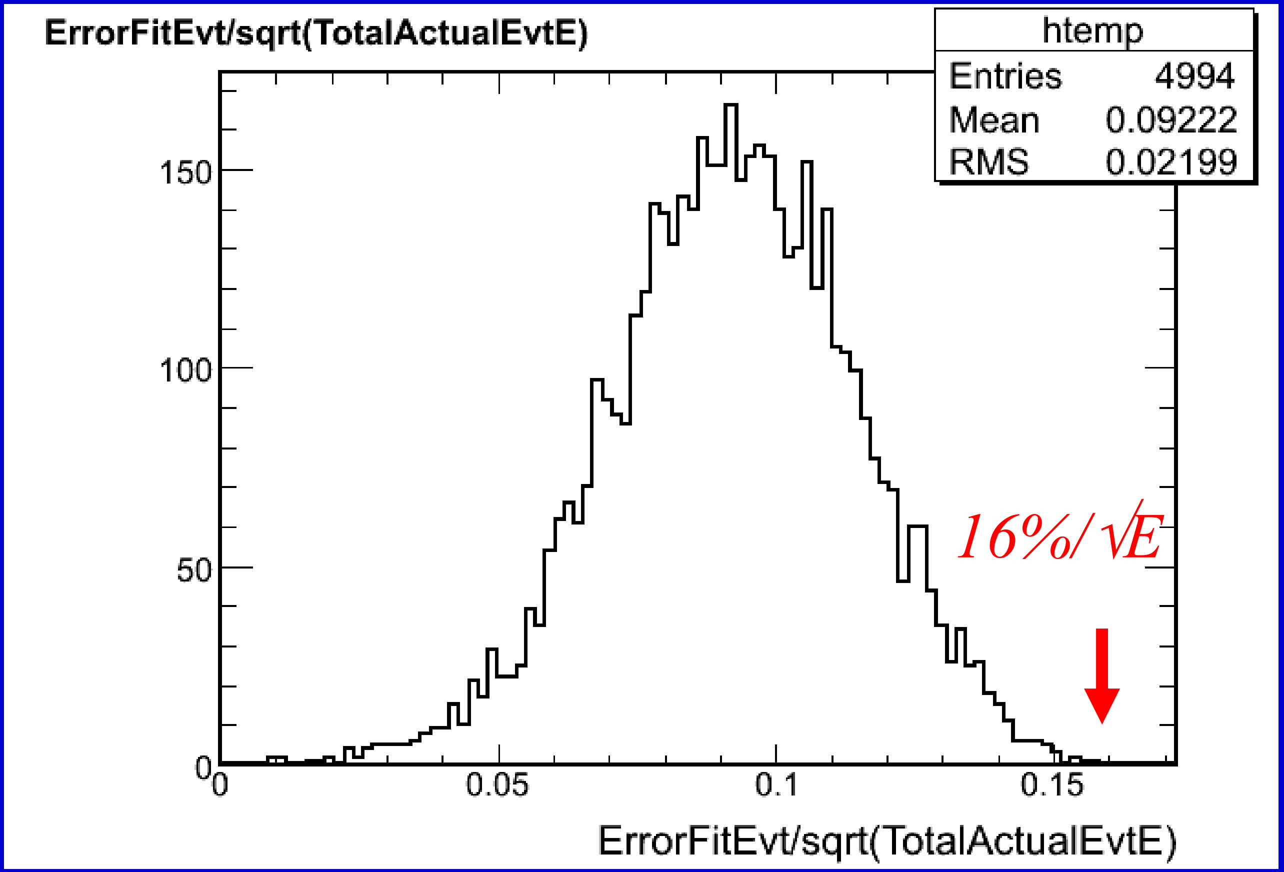
Example (p_{fit} = 0.5%)

	E1 (GeV)	*E2 (GeV)*
G	*2.8*	*1.2*
M	*2.5*	*2.0*
F	*1.9*	*1.7*

Z^0

16%/ √E, 0.5mr, perfect pairing

Calculate error on the sum of the fitted π^0 energies and scale stochastically



Potential of energy resolution of around 9.2%/ √E on ***average***

Next Steps

- Finalize current studies and complete write-up.
- Implement on simulated single π^0's
  - Need appropriate clustering, calibrated ECAL and errors.
  - Expect to put some emphasis on low energy photons.
  - While the ILD ECAL is not over-designed for this application, doing "real" simulation studies again will be an important complement to this more conceptual work, and will enable studies in the PFA framework.
  - To get the full benefit – need some more segmented ECAL layers (eg. MAPS or analog Si-strips). MAPS based ECAL layers are well matched to this application !
- Re-visit (and write up) "matching problem" – pairing up photons in hadronic events.
  - (Old results $16\%/\sqrt{E} \rightarrow 12\%/\sqrt{E}$) (9.4%)

Conclusions and Outlook

- Kinematic fitting works
  - Detector designs should take advantage.

- Excellent angular resolution for photons can lead to much improved resolution on EM component of hadronic jets (and knowledge of the error).

- Measuring very well some jets (those without neutral hadrons), and knowing the resolution, will be advantageous in some physics analyses.

Backup Slides

π^0 mass resolution

- Can show that for $\sigma_E/E = c_1/\sqrt{E}$ that
  $\Delta m/m = \mathbf{c_1} /\sqrt{[(1-a^2) E_{\pi 0}]} \oplus 3.70\, \mathbf{\Delta\psi_{12}} E_{\pi 0} \sqrt{(\beta^2-a^2)}$
  where $a = \beta \cos\theta^* = (E_1-E_2)/E_{\pi 0}$

So the mass resolution has 2 terms :

i) depending on the EM energy resolution (c_1)

ii) depending on the opening angle resolution (**$\Delta\psi_{12}$**)

The relative importance of each depends on ($E_{\pi 0}$, a)

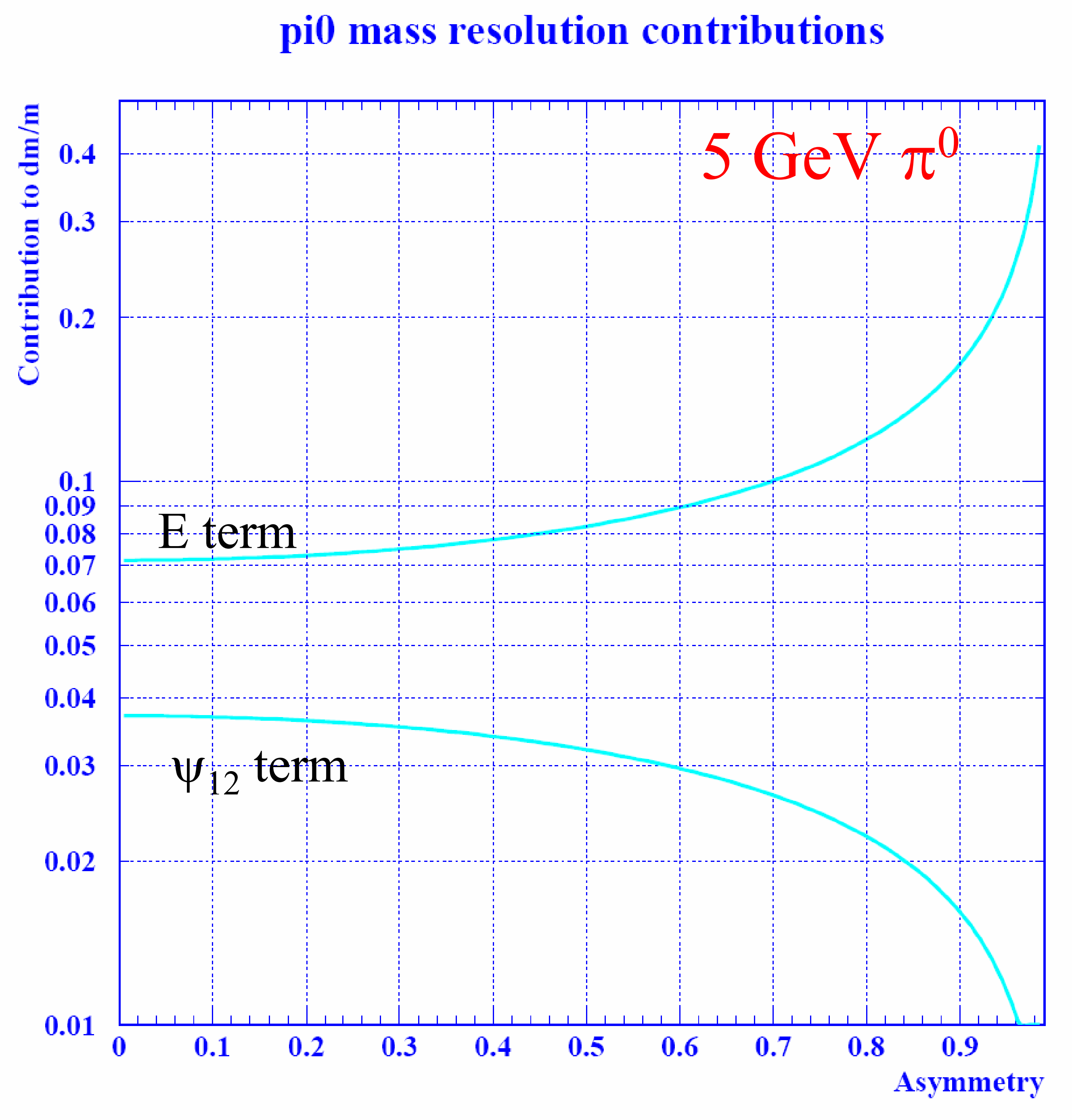
π^0 mass resolution

Plots assume:

$c_1 = 0.16$ (SiD)

$\Delta\psi_{12} = 2$ mrad

For these detector resolutions, 5 GeV π^0 mass resolution dominated by the E term



Recent Improvements

- Blobel numerical fitter in DP in addition to analytic fit (both F77 for now)
  - consistent
- Technical details
  - $\cos\theta^* = (1/\beta)\,(E_1 - E_2)\,/\,E_{\pi^0}$
  - Error truncation for low energies : avoid –ve energies …
  - Using simulated error rather than measured error
  - Now have ***perfect*** probability and pull distributions
- Error propagation after kinematic fit
  - Demonstration that for each π^0 in the event, we could not only improve the π^0 energy resolution but would also **know the error**.

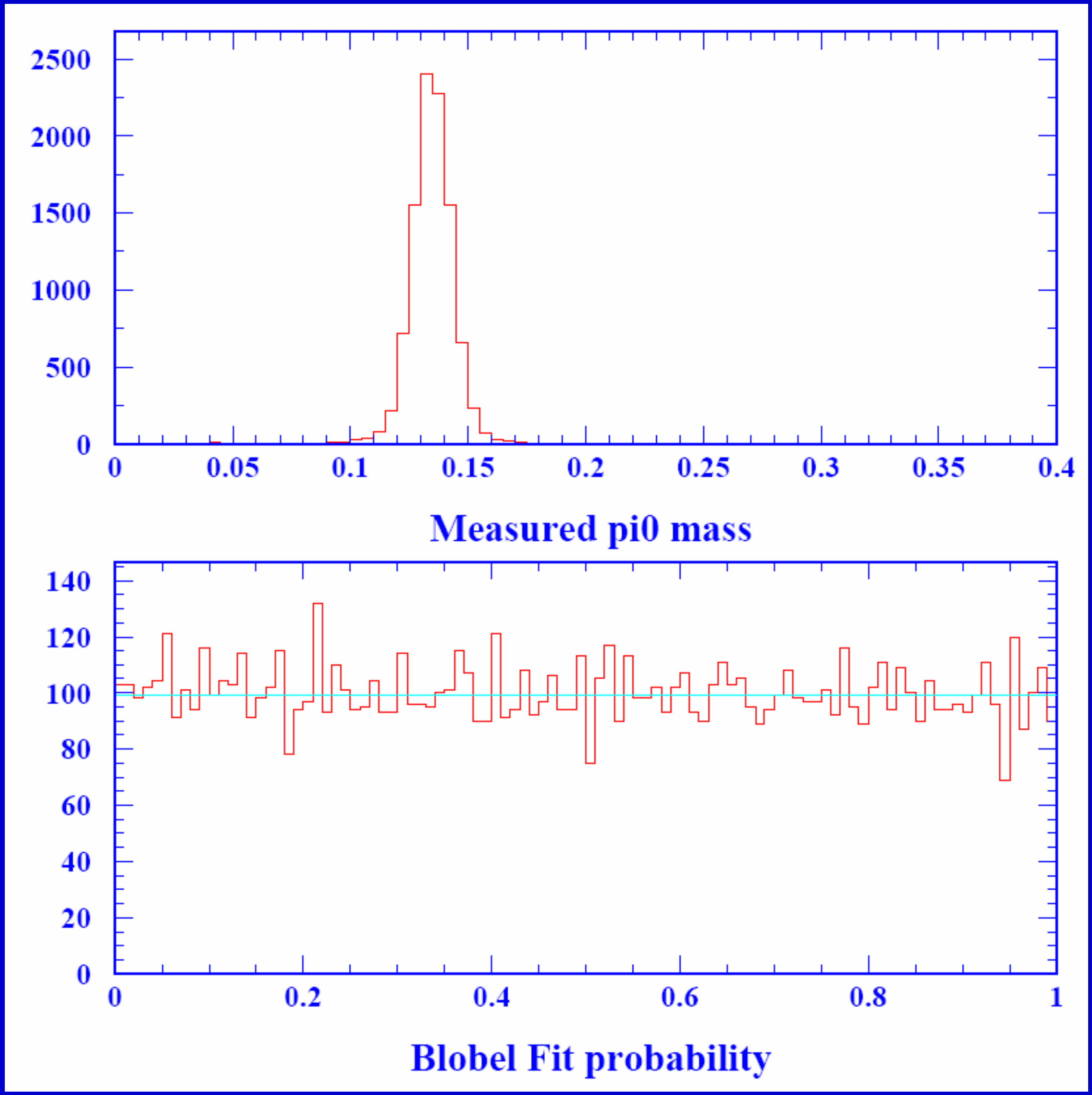
20 GeV π^0

Use single π^0 toy MC with Gaussian smearing for studies.

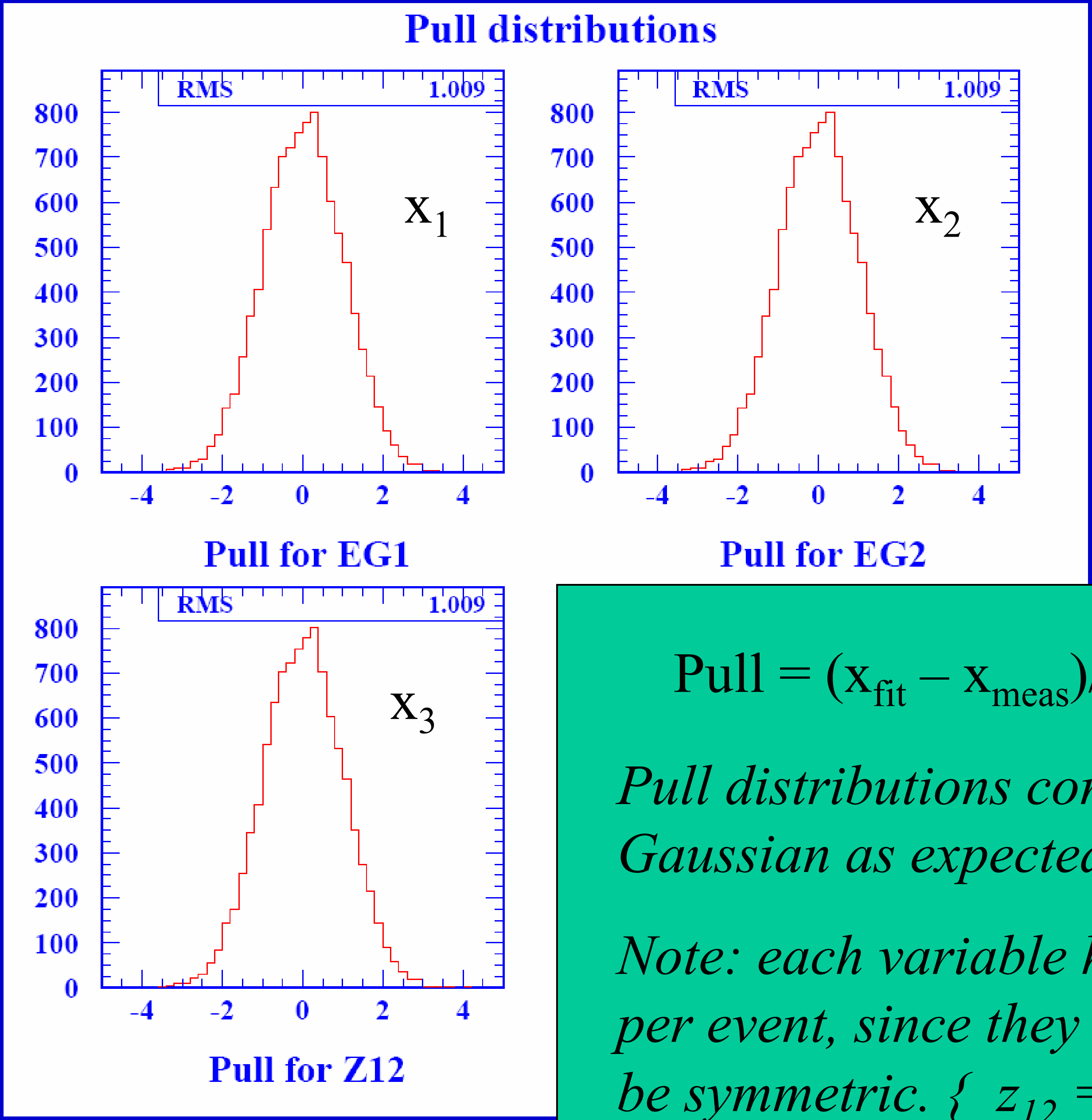
Energy resolution per photon $=16\%/\sqrt{E}$.

Error on ψ_{12}=0.5 mrad.

These resolutions used unless otherwise stated.



A rare thing: a really flat probability distribution !!!

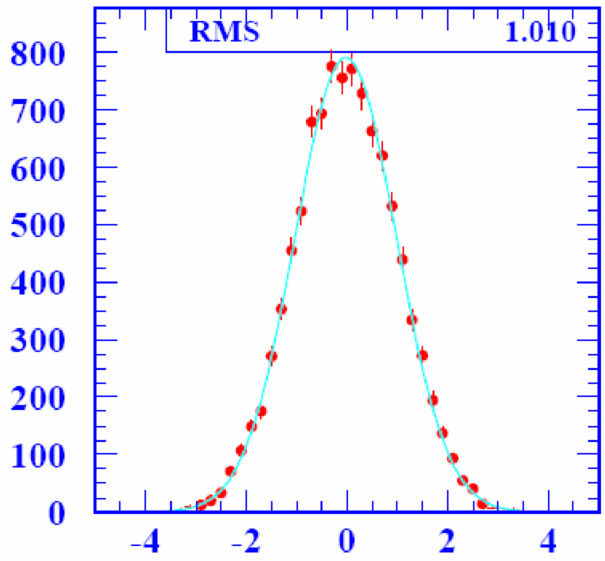


$$\text{Pull} = (x_{fit} - x_{meas})/\sqrt{(\sigma_{meas}^2 - \sigma_{fit}^2)}$$

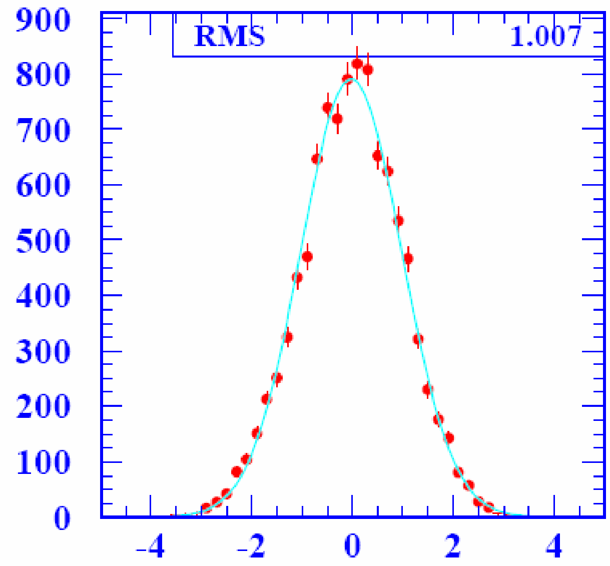
Pull distributions consistent with unit Gaussian as expected.

Note: each variable has an identical pull per event, since they were constructed to be symmetric. { $z_{12} = 2(1-\cos\psi_{12})$ }

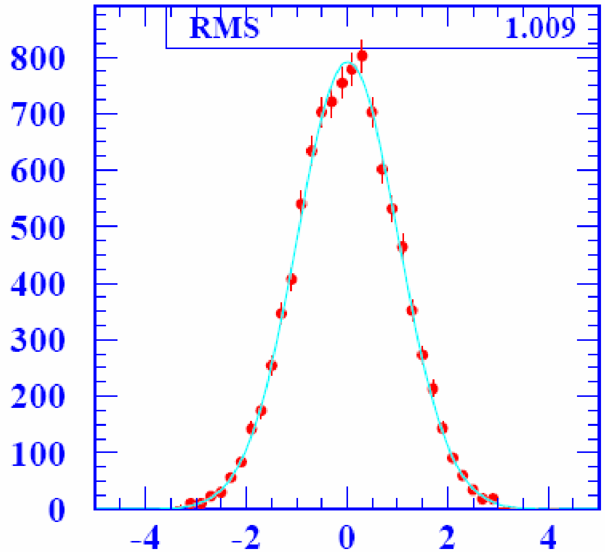
Pull distributions



Measured pi0 energy pull cf gen



Fitted pi0 energy pull cf gen



Fitted pi0 energy Pull cf measured

=> You should also be able to believe the errors on the fitted energies of each π^0

3. Results on π^0 Energy Resolution Improvement

For the Proof of Principle study there are:

Two relevant π^0 kinematic parameters:

i) E (π^0)

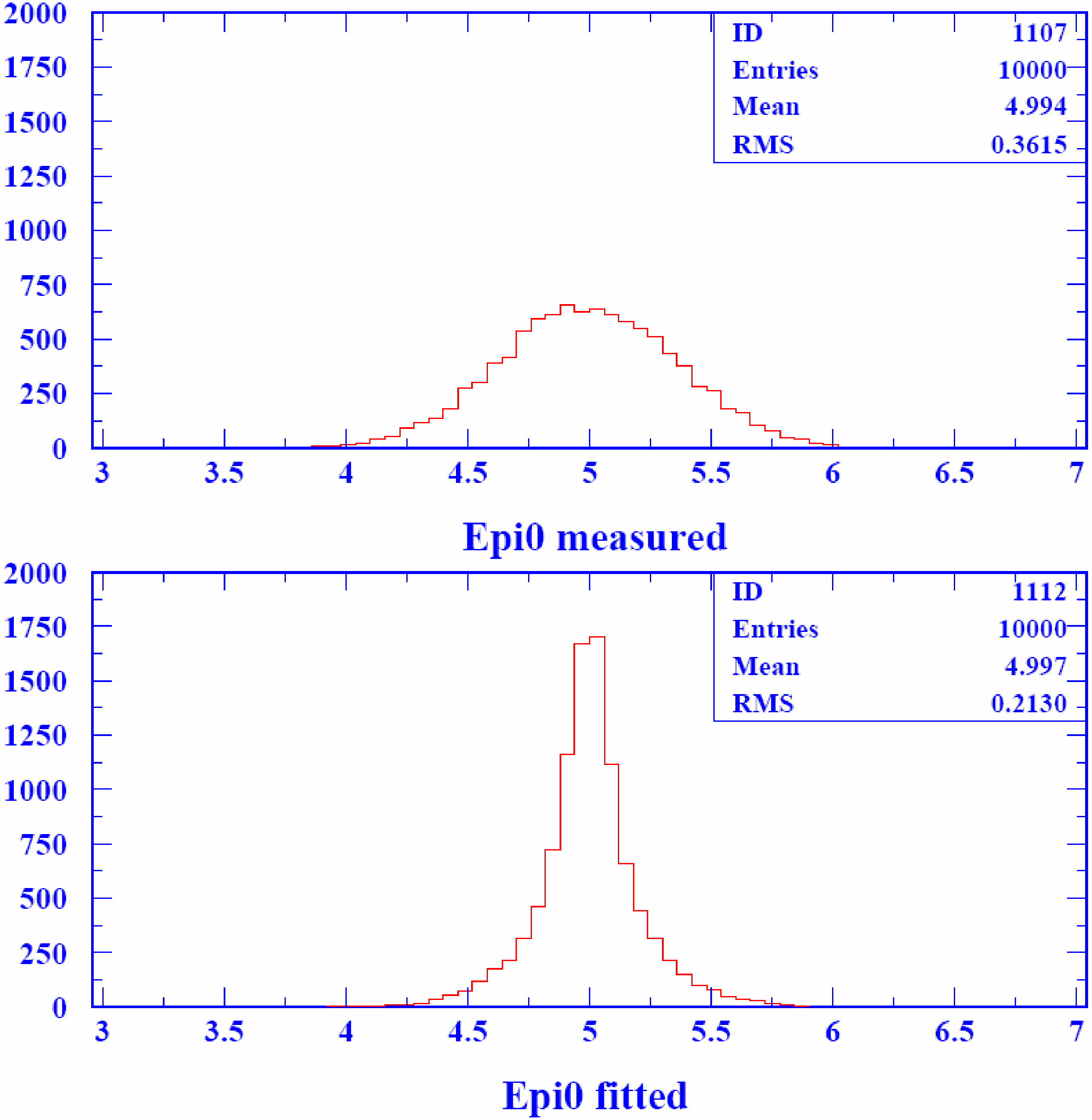
ii) cosθ^* (cosine of CM decay angle)

And two relevant detector parameters:

i) Photon fractional energy resolution ($\Delta E/E$)

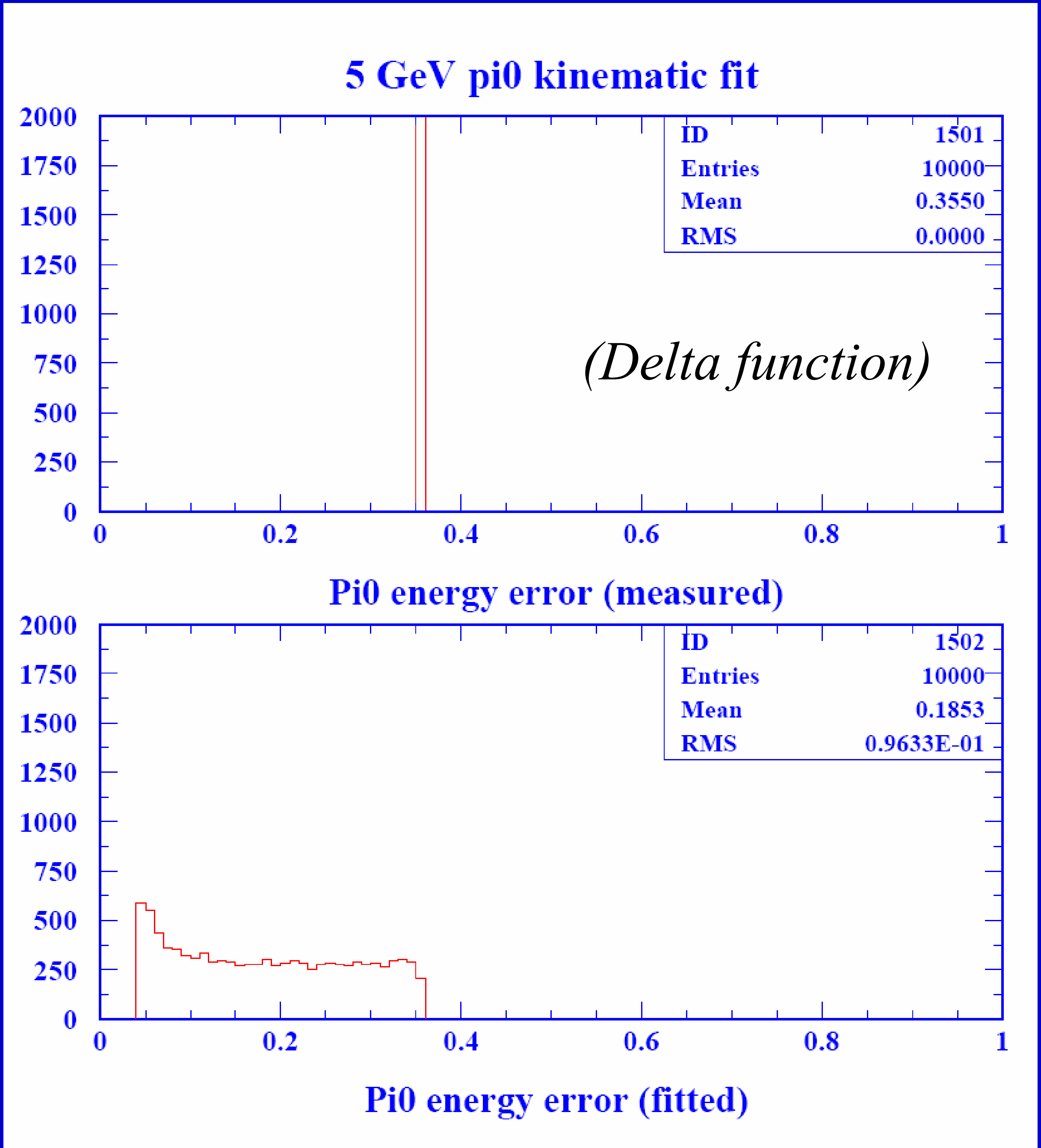
ii) Opening angle resolution ($\Delta\gamma$ [illegible])

5 GeV pi0 kinematic fit



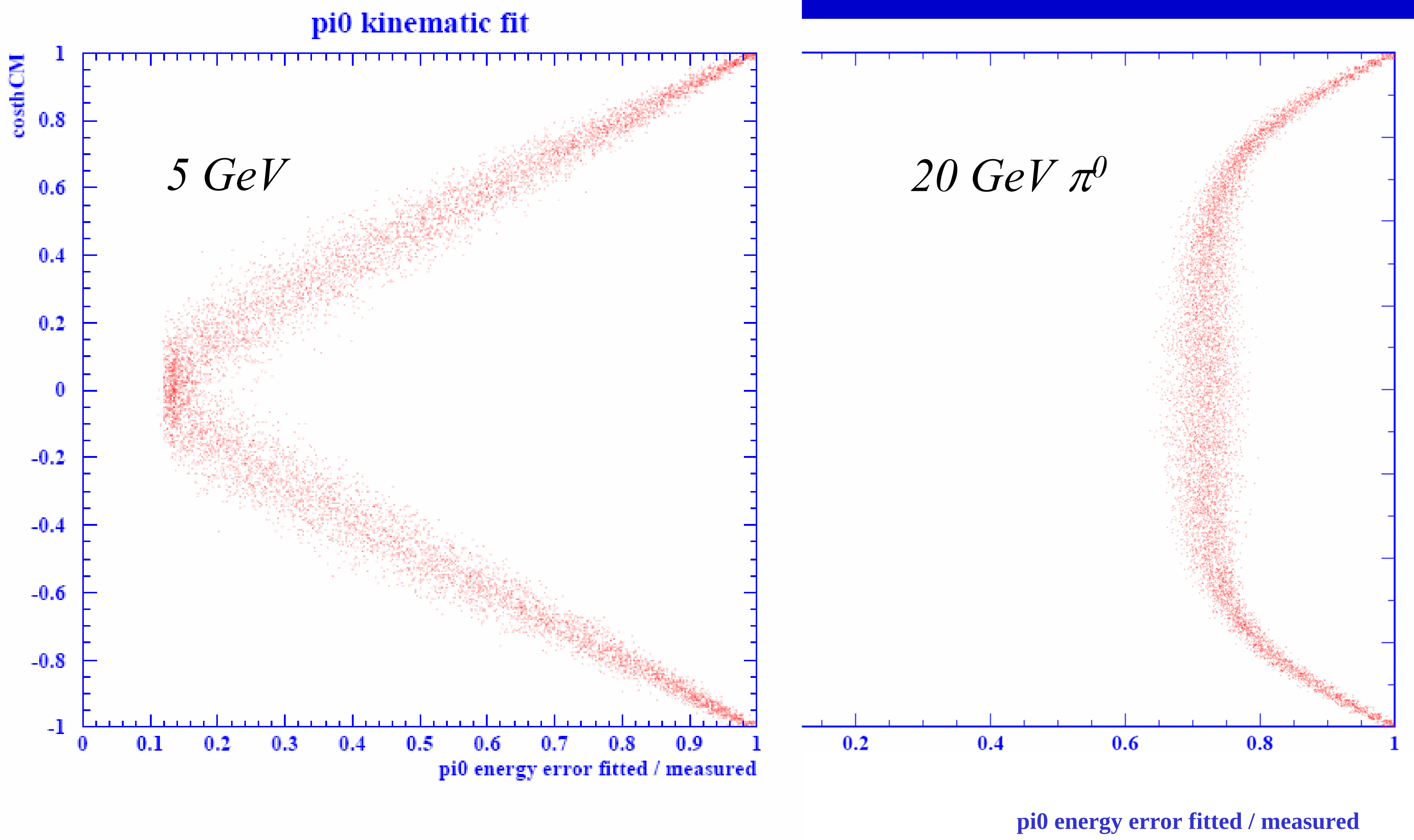
DRAMATIC IMPROVEMENT

But this plot is not really a good representation of what is going on.

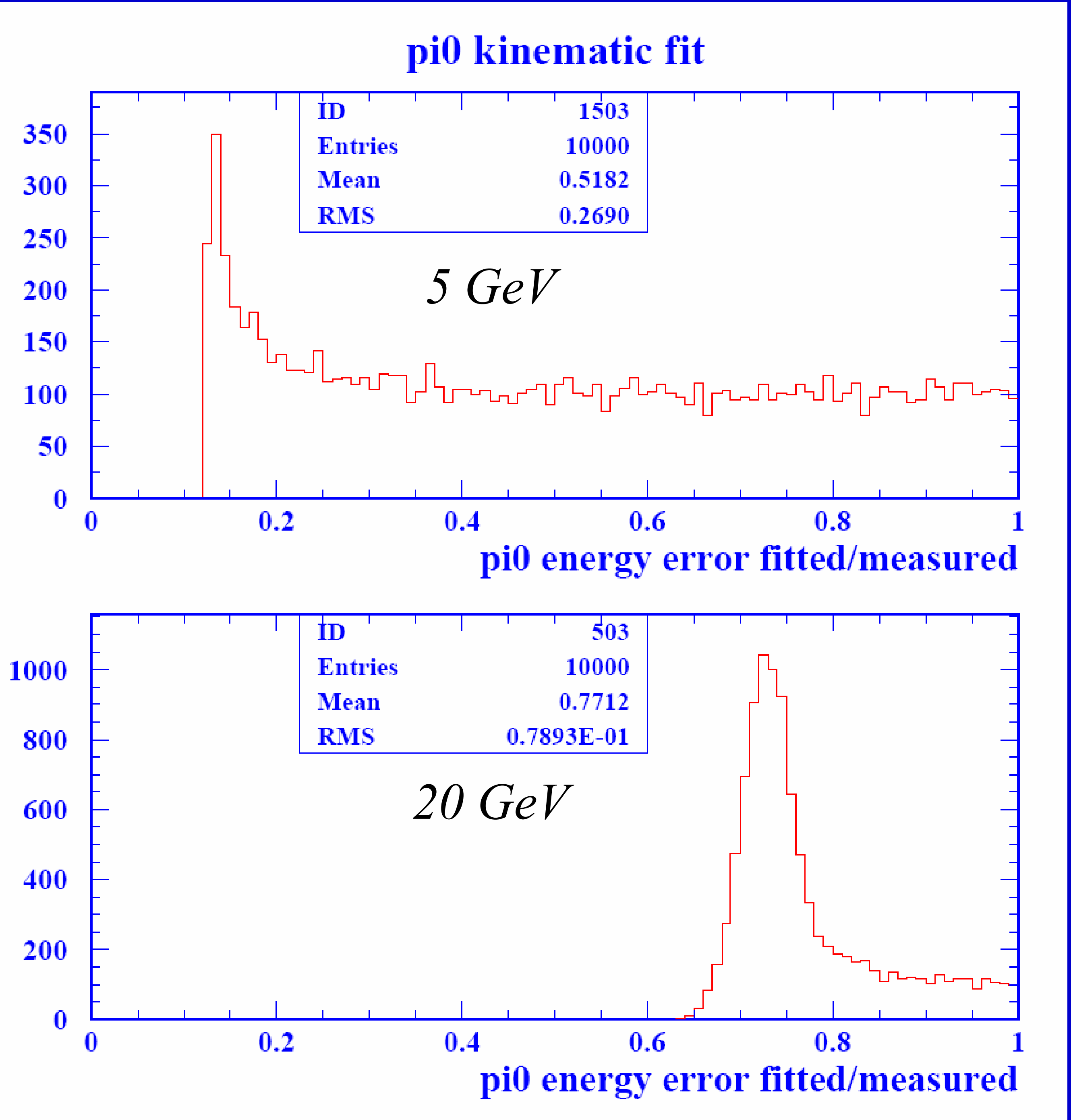


From now on, will use the π^0 energy error ratio (fitted/measured) as the estimator of the improvement.

Call this the improvement ratio.



Very strong dependence of fit error on cosθ*.
Symmetric decay (cosθ*=0) is best

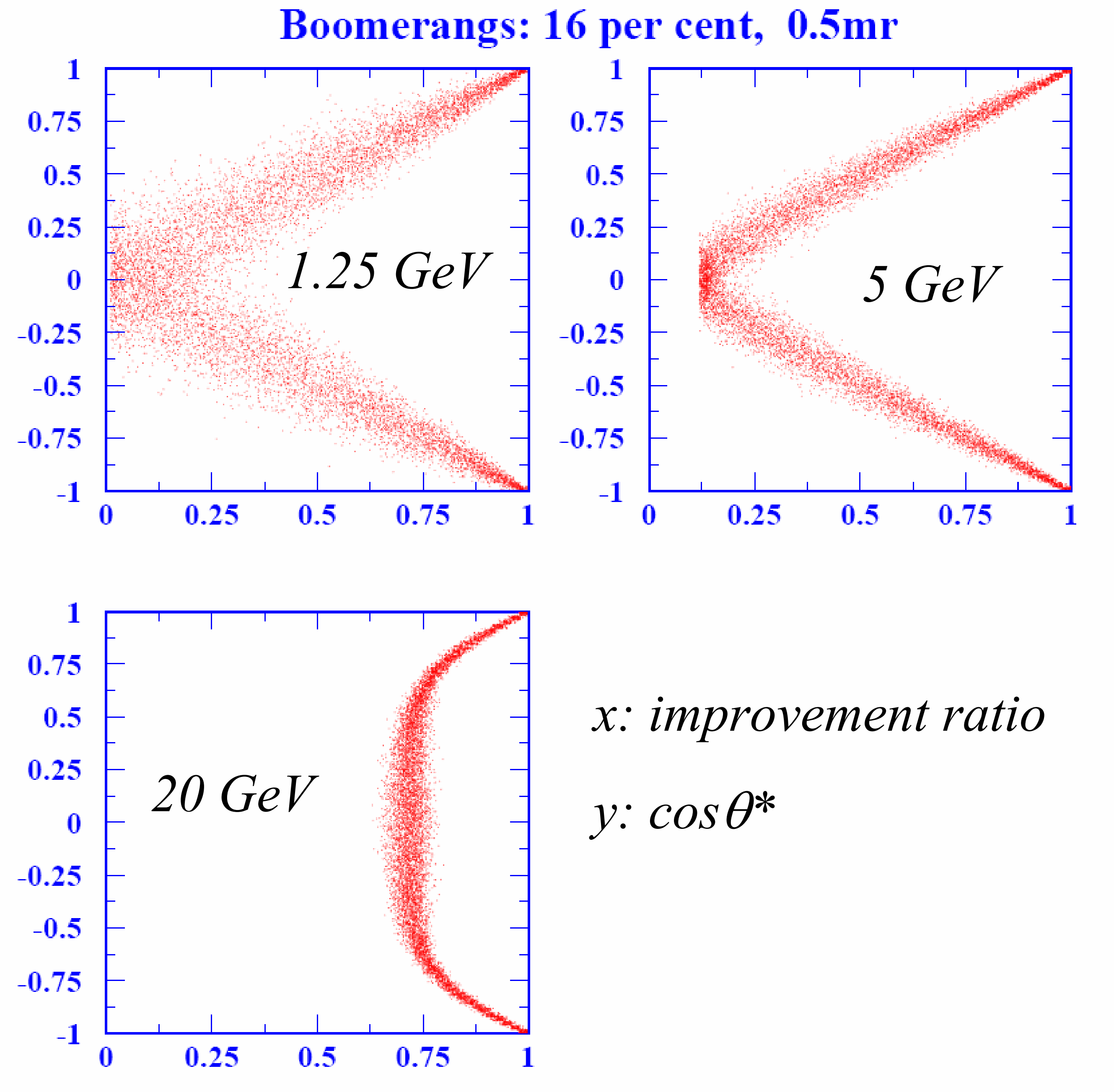


Improvement by up to a factor of 7 !

On average, factor of 2.

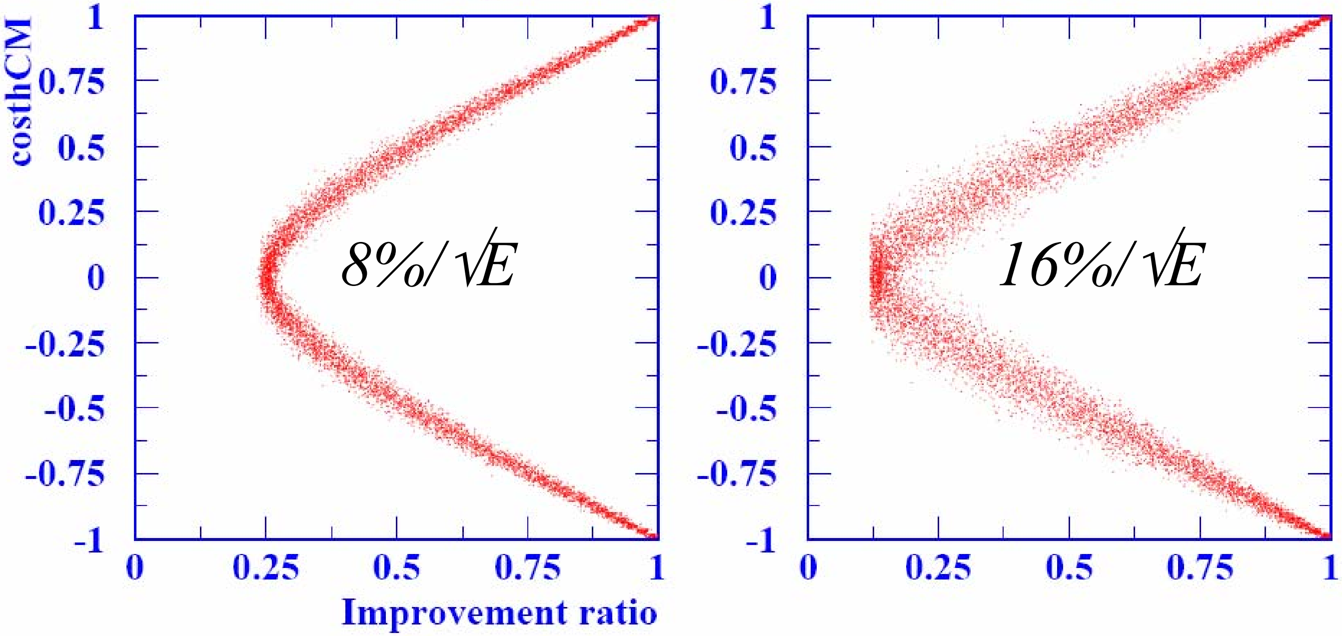
Improves by a factor of 1.3 on average.

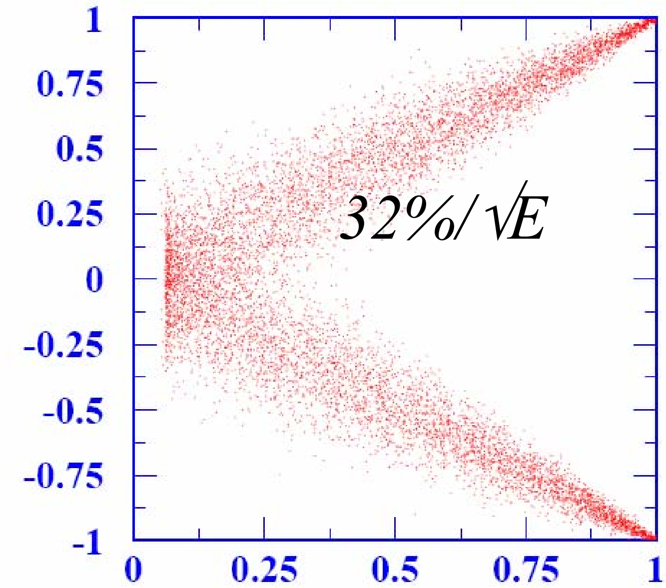
Dependence on π^0 energy



5 GeV π^0

Varying Energy Resolution 11,21,31





Improvement ratio (x-projection) ***DOES*** *depend on Energy resolution (for this π^0)*

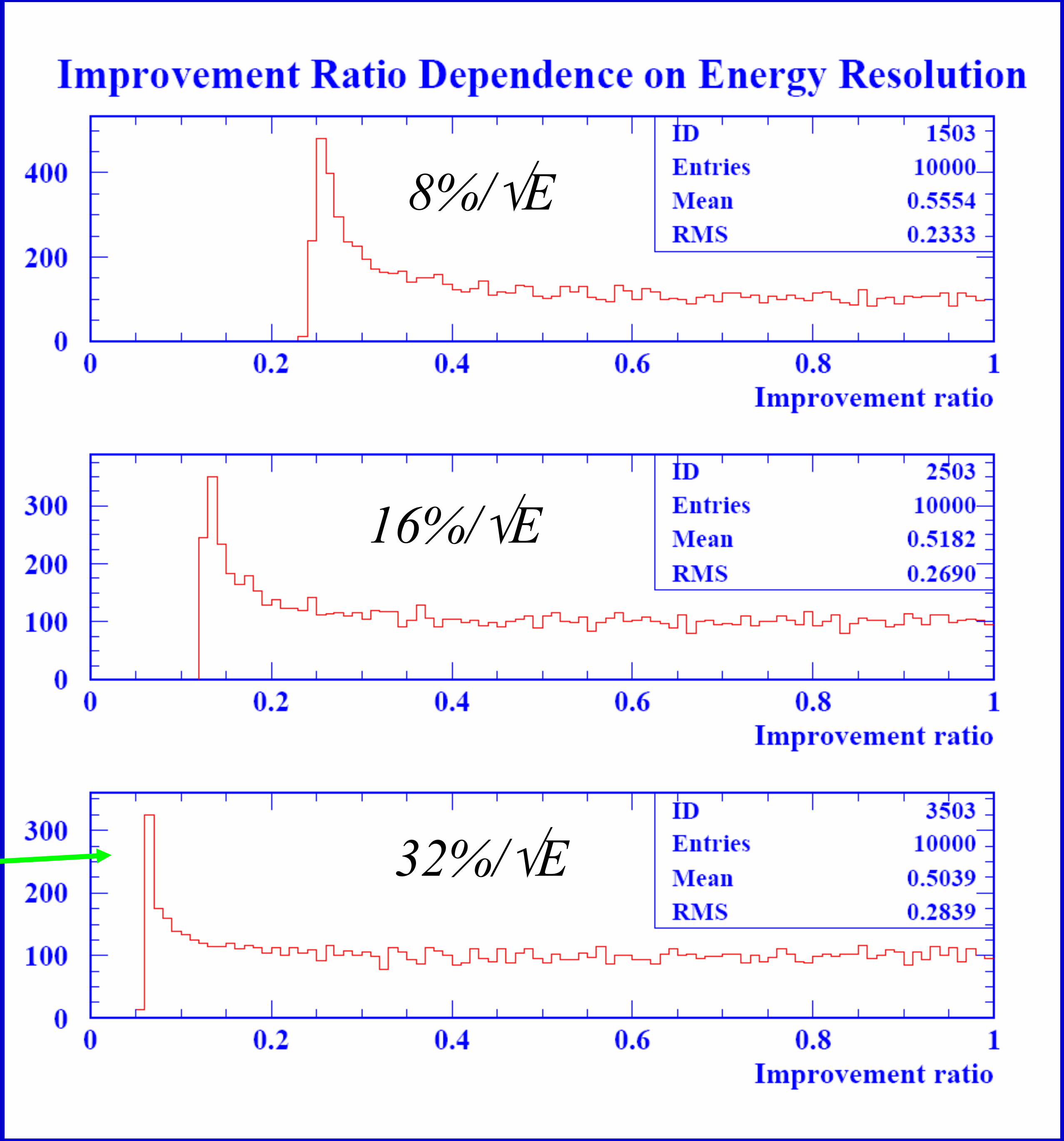
- But on average the dependence is only weak (see next slide)

This slide has been corrected from that presented at Vancouver

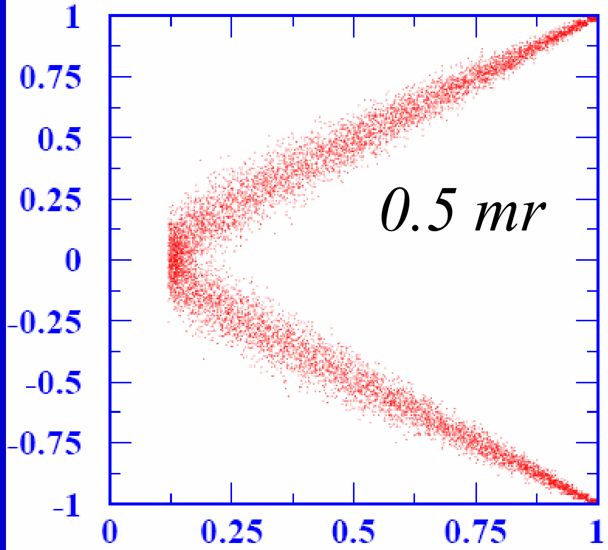
5 GeV π^0

Average improvement factor not highly dependent on energy resolution.

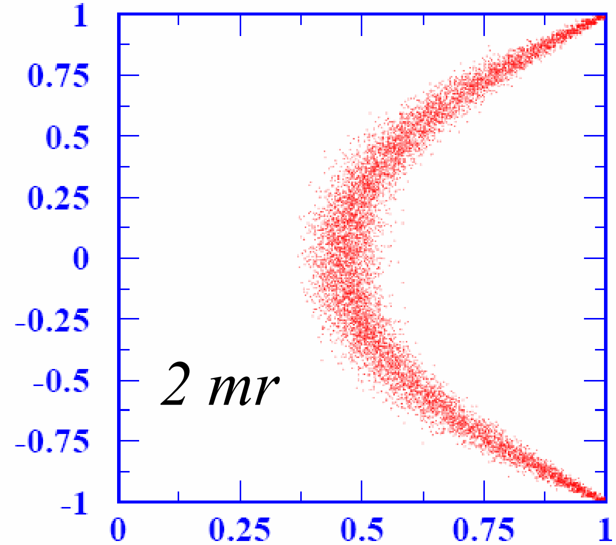
BUT the maximum possible improvements increase as the energy resolution is degraded.



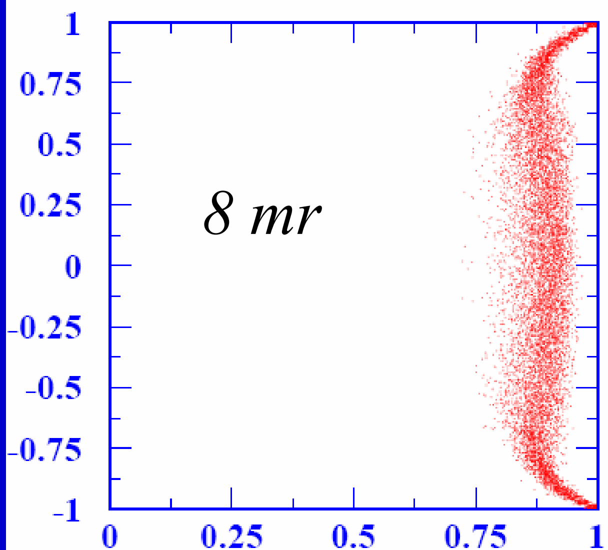
5 GeV pi0, 16%, vary ang resolution



pi0 energy error ratio vs costhcm



pi0 energy error ratio vs costhcm

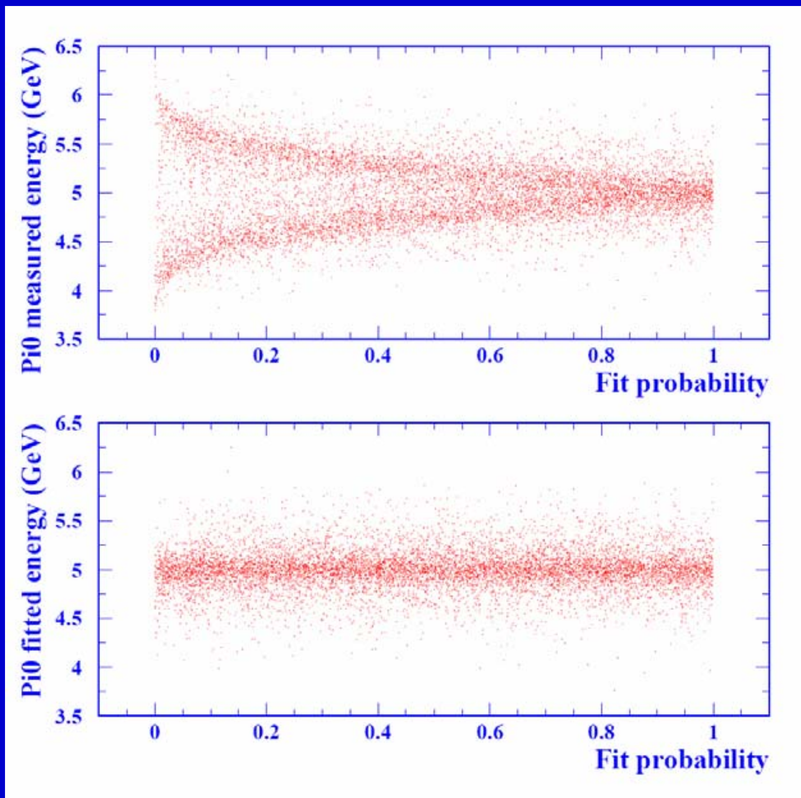


pi0 energy error ratio vs costhcm

Angular resolution very important ...

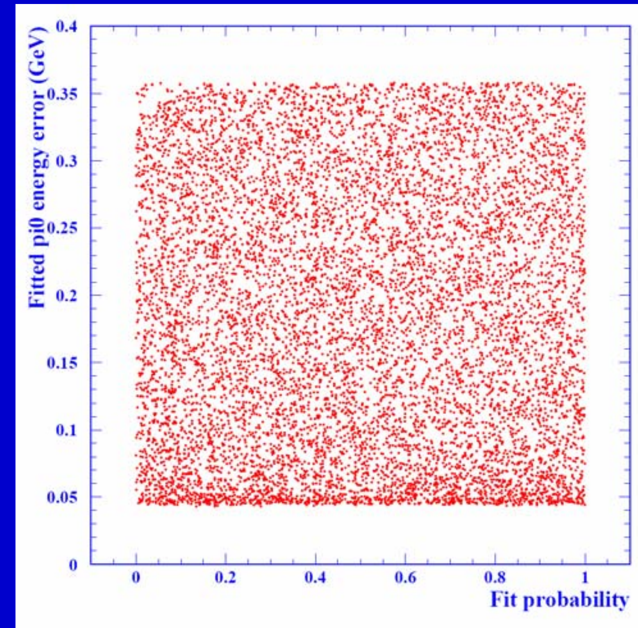
What's going on ?

5 GeV π^0, c_1=16%, $\Delta\psi_{12}$=0.5mr

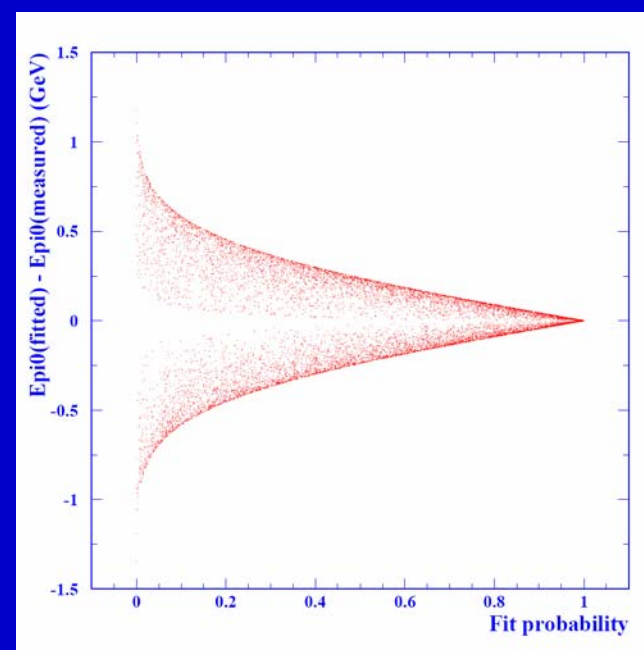


E_{π^0} changes most when p_{fit} small.

(NB the constraint is correct, so low p_{fit} corresponds to π^0's where typically the energy has fluctuated substantially)



Error on π^0 energy is independent of p_{fit}



Hard edges correspond to low $|\cos\theta^*|$

Kinematic Fitting Summary

- Proof of principle of kinematic fit for π^0 reconstruction done.
  - Kinematic fit infrastructure now a solid foundation.
  - Well understood errors on each π^0.
- Potential for a factor of two improvement in the energy resolution of the EM component of hadronic jets.

4. Towards applying to hadronic jets

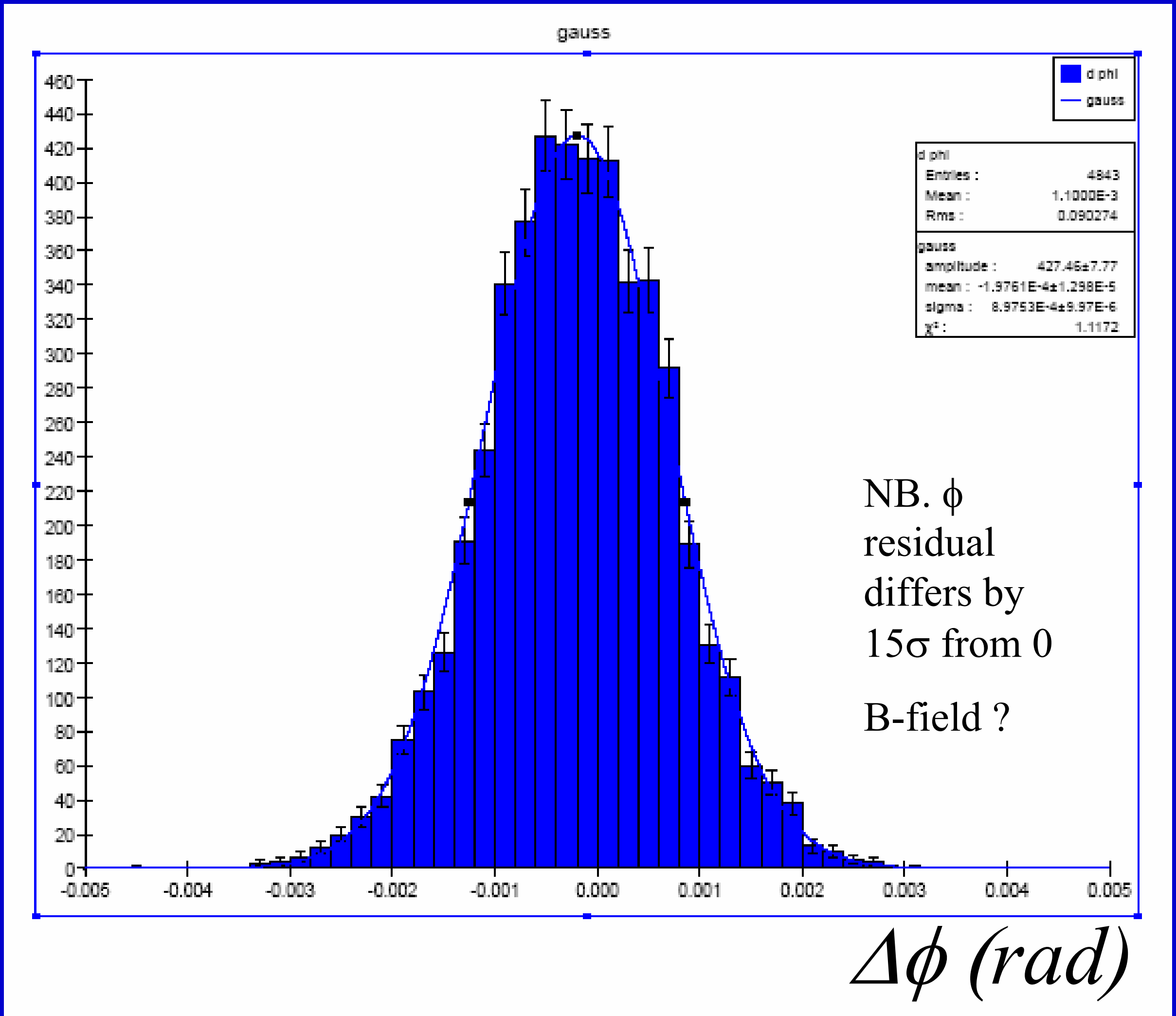
- Detector response
- Characterize the multi-photon issues in Z $\rightarrow$ uu, dd, ss events.
  - Define prompt photons as originating within 10 cm of the origin
    - (NB differs from standard $c\tau < 10$ cm definition)

Angular Resolution Studies

5 GeV photon at 90°, sidmay05 detector (4 mm pixels, R=1.27m)

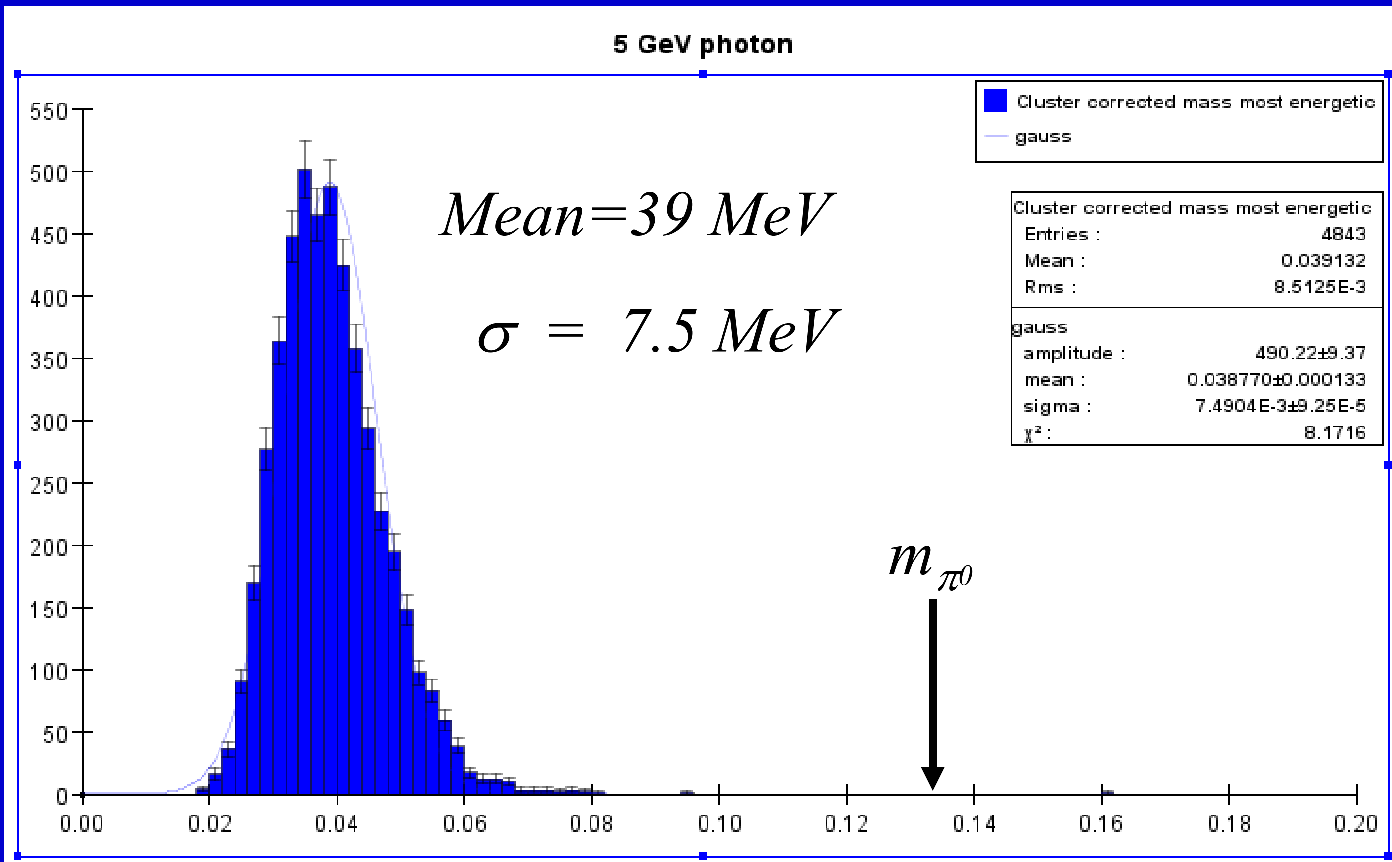
Phi resolution of 0.9 mrad ***just*** using cluster CoG.

=> θ_{12} resolution of 2 mrad is easily achievable for spatially resolved photons.



NB. Previous study (see backup slide), shows that a factor of 5 improvement in resolution is possible at fixed R using longitudinally weighted "track-fit".

Cluster Mass for Photons



Cluster Mass (GeV)

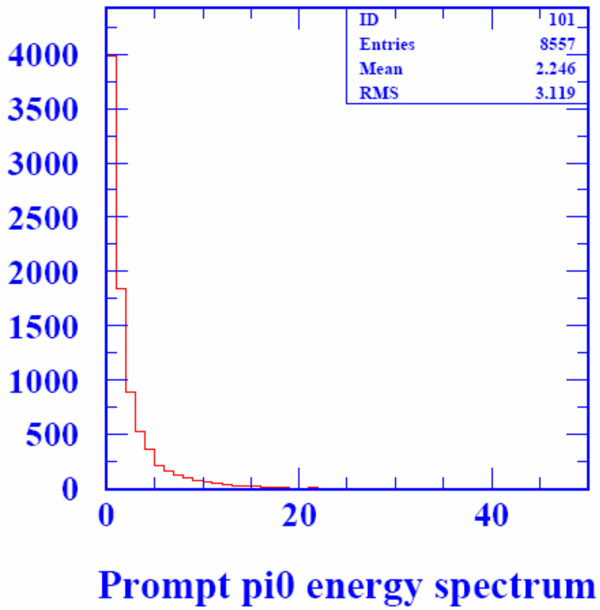
Of course, photons actually have a mass of zero.

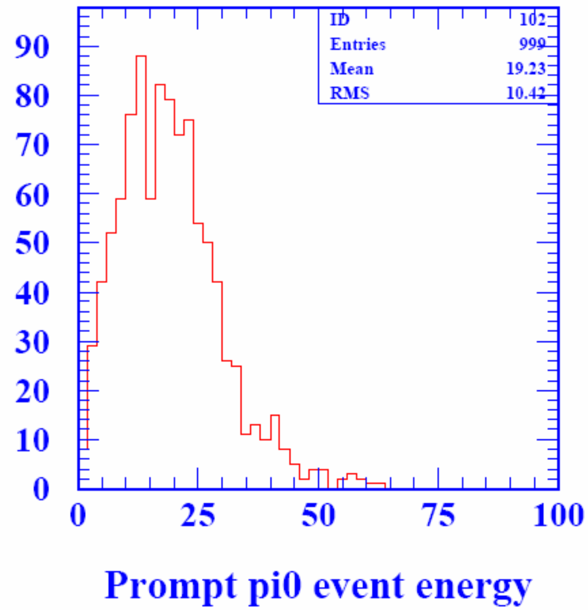
The transverse spread of the shower leads to a non-zero cluster mass calculated from each cell.

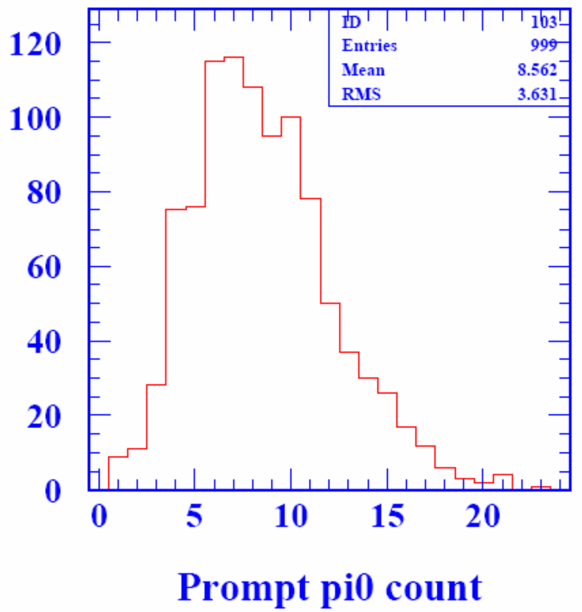
Use to distinguish single photons from merged π^0's.

Performance depends on detector design (R, R_M, B, cell-size, …)

Z to uu, dd, ss at 91 GeV

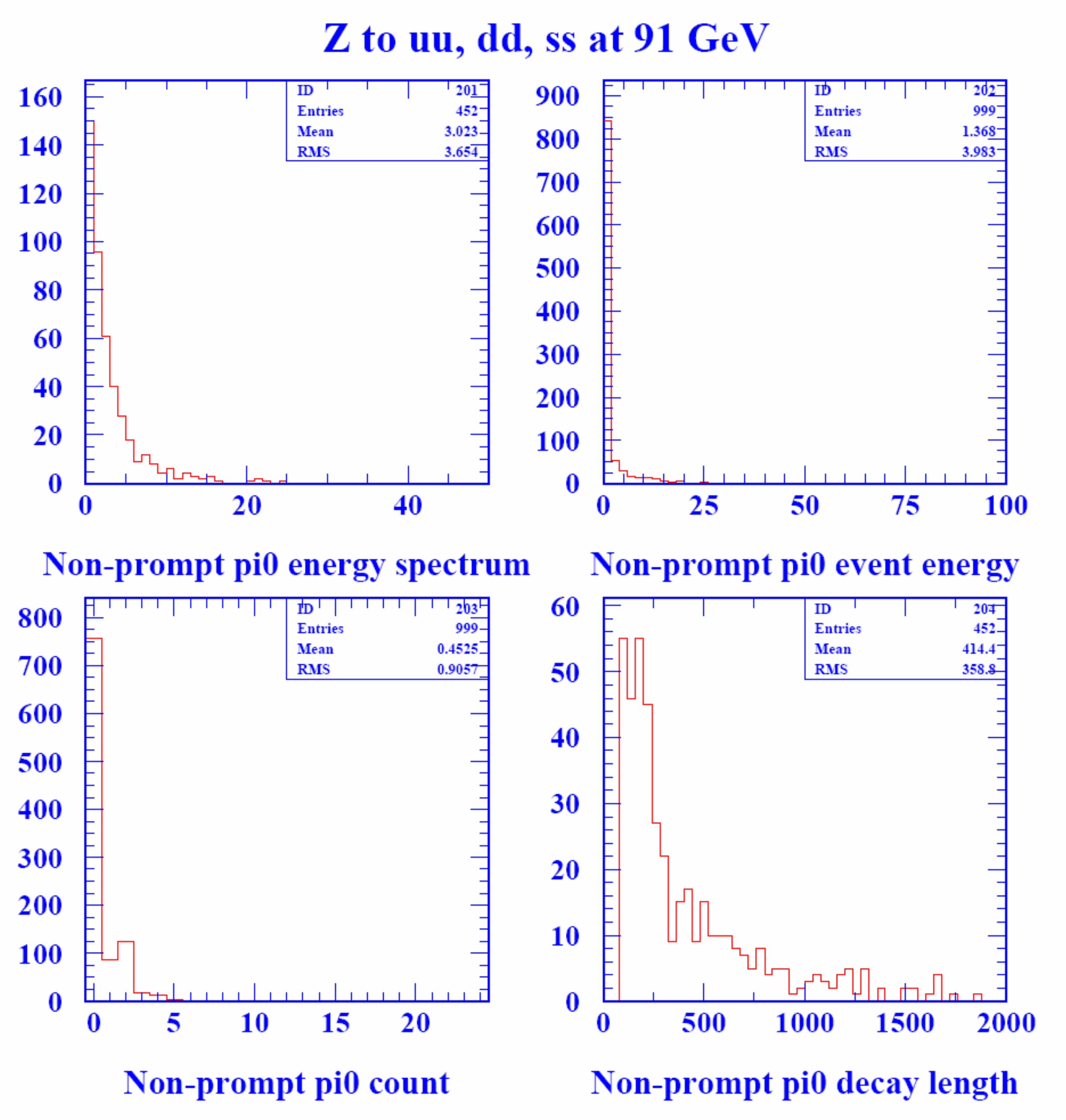






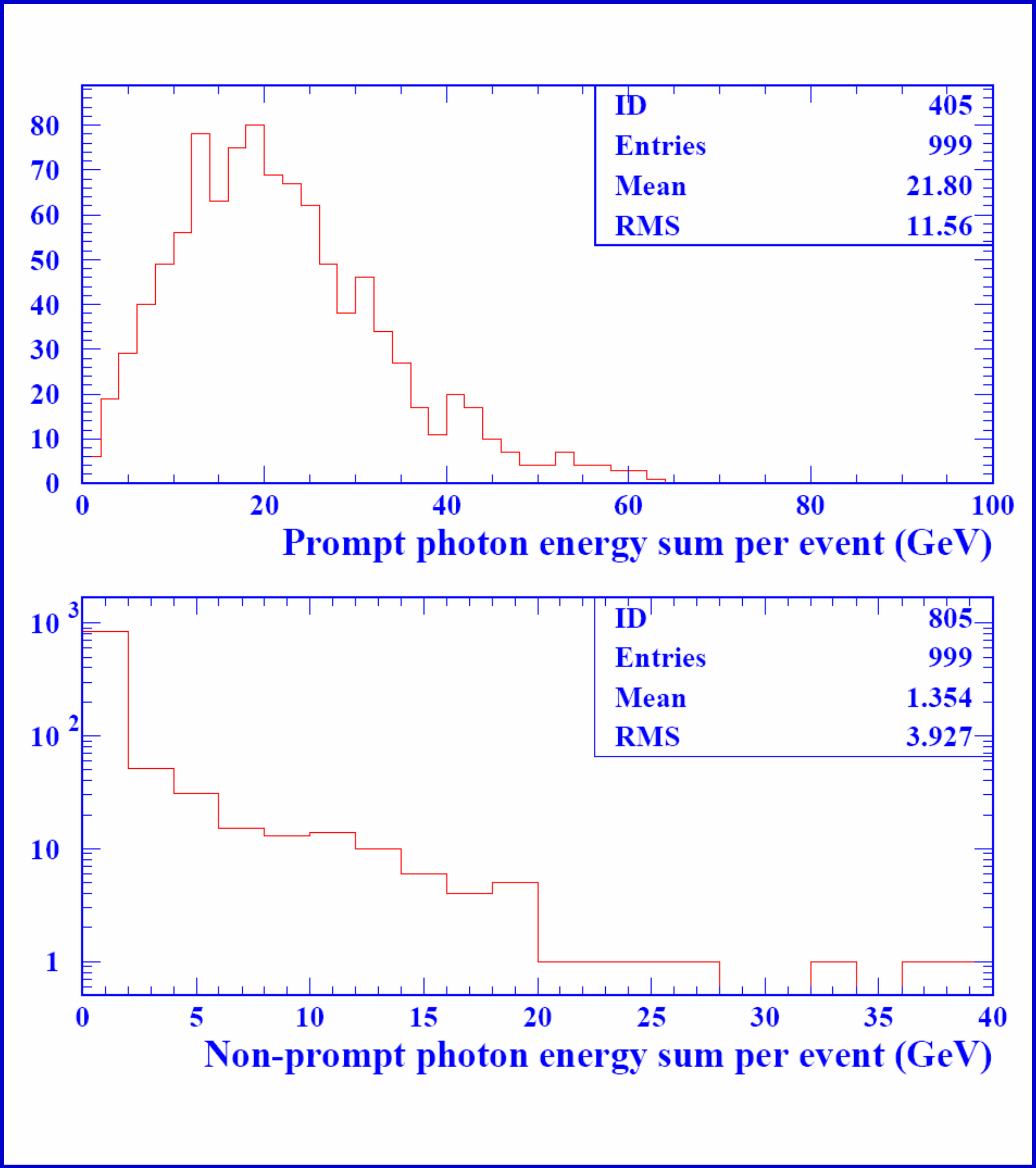
On average 19.2 GeV (21.0%)

NB generator has ISR and beamsstrahlung turned off.



On average, 1.4 GeV (1.5%)

Photon Accounting



cf 19.2 GeV from prompt π^0

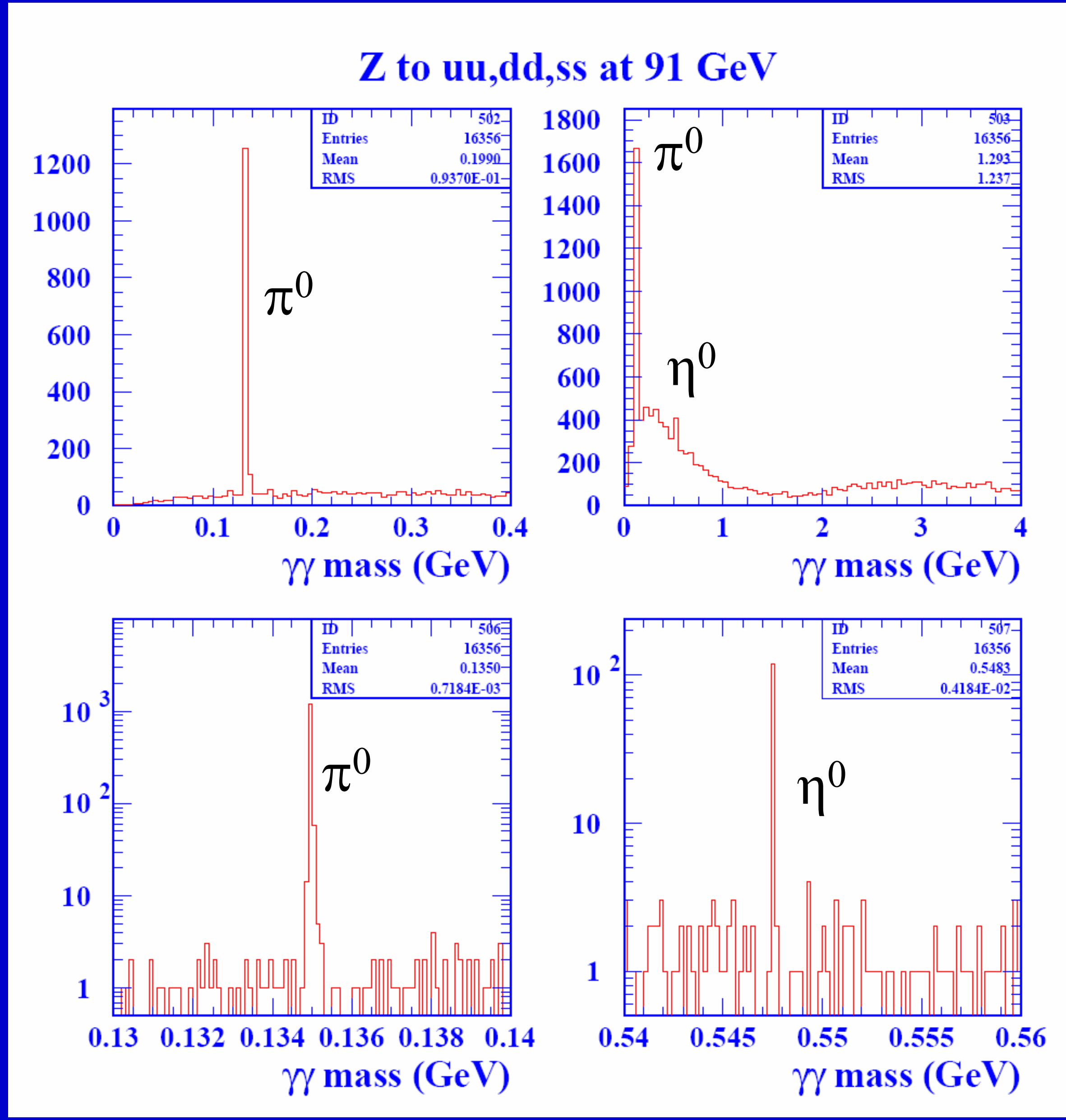
Intrinsic *prompt* photon combinatorial background in $m_{\gamma\gamma}$ distribution assuming perfect resolution, and requiring $E_{\gamma} > 1$ GeV.

With decent resolution, the combinatoric background looks manageable:

0.09 combinations / 10 MeV/event (π^0),

0.06 combinations/10 MeV/event (η).

Especially if one adopts a strategy of finding the most energetic and/or symmetric DK ones first.



Next step: play with some algorithms

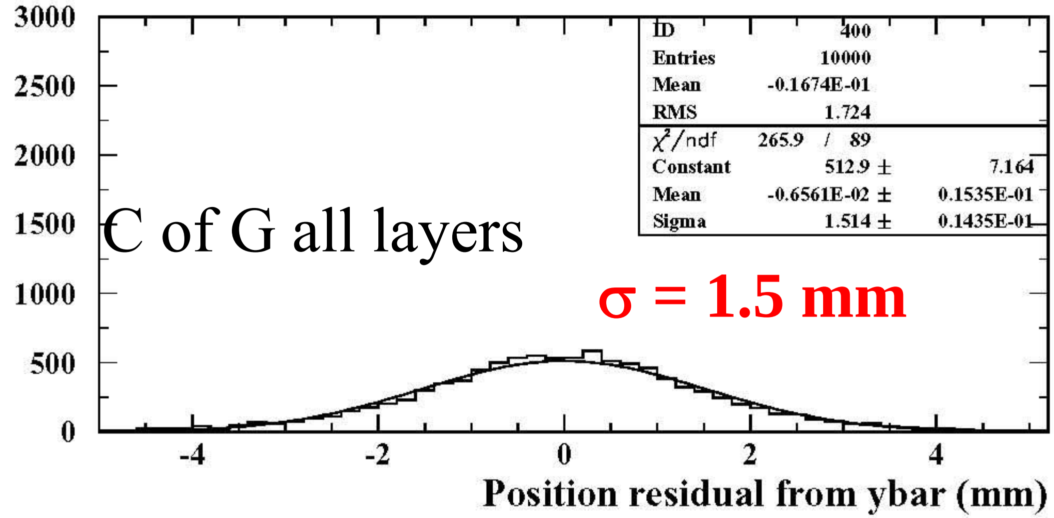
Position resolution from simple fit

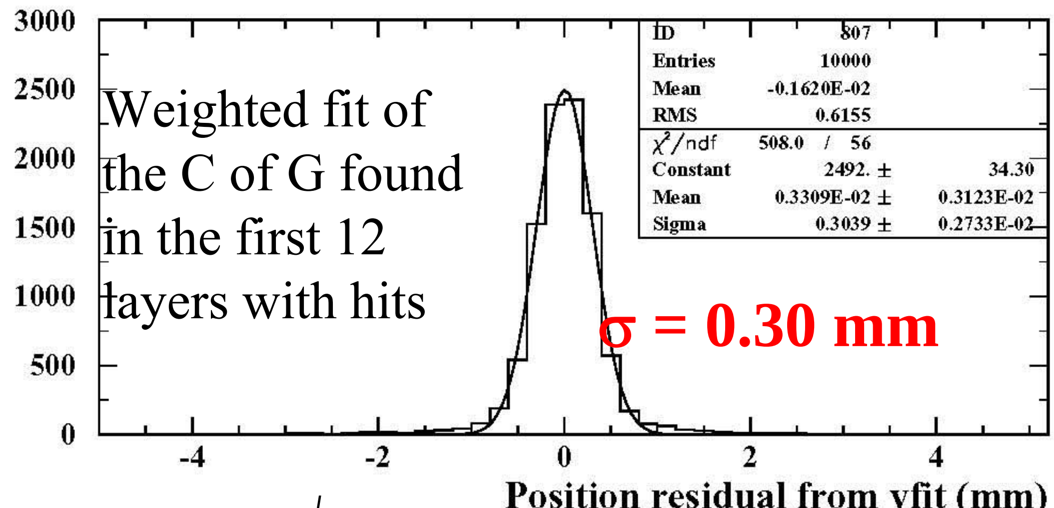
Neglect layer 0 (albedo)

Using the first 12 layers with hits with E>180 keV, combine the measured C of G from each layer using a least-squares fit (errors varying from 0.32mm to 4.4mm). Iteratively drop up to 5 layers in the "track fit".

*Position resolution does indeed improve by a factor of **5** in a realistic 100% efficient algorithm!*

1 GeV photon, G4 study (GWW)



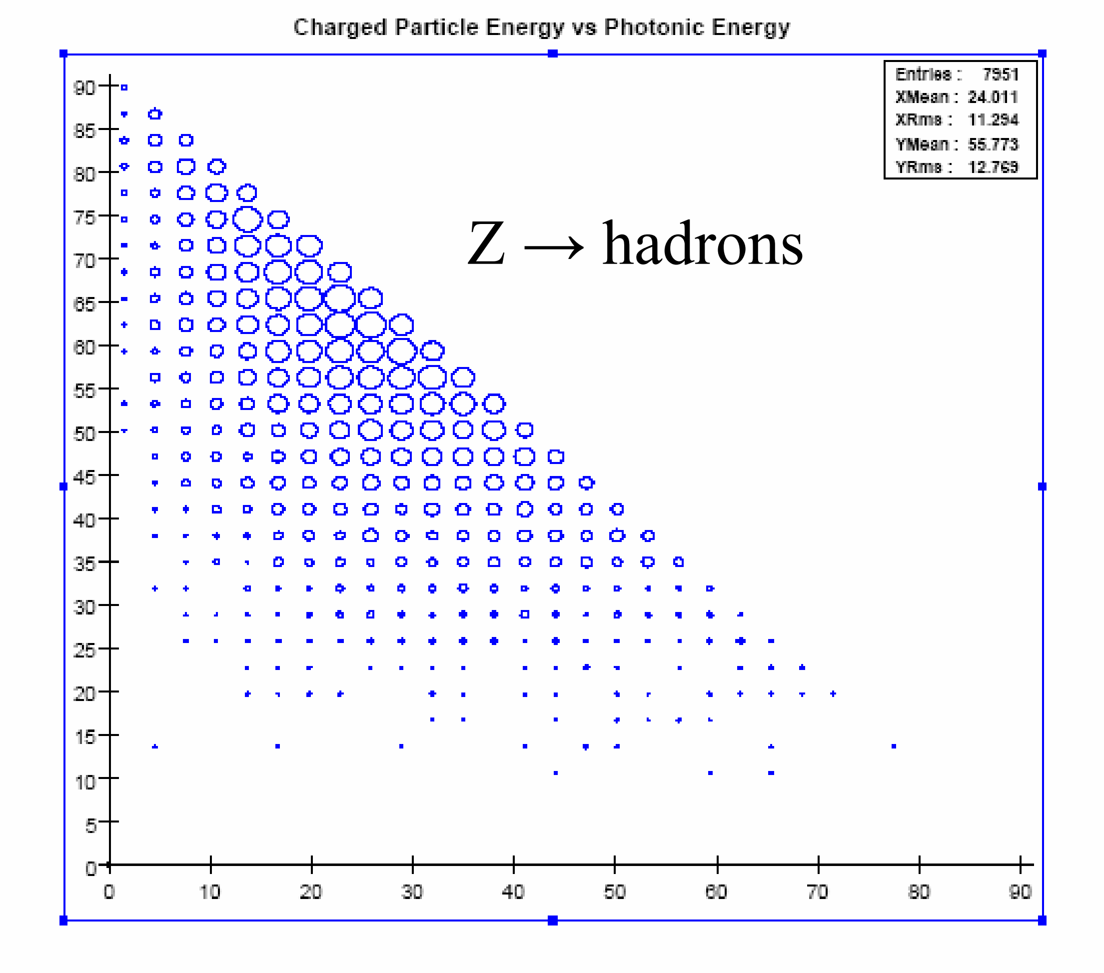


Still just d/√12 !

PFA "Dalitz" Plot

Also see: http://heplx3.phsx.ku.edu/~graham/lcws05_slacconf_gwwilson.pdf

"On Evaluating the Calorimetry Performance of Detector Design Concepts", for an alternative detector-based view of what we need to be doing.



On average, photonic energy only about 30%, but often much greater.

γ, π^0, η^0 rates measured at LEP

	Experimental results				JETSET 7.4	HERWIG 5.9
	OPAL	ALEPH [6]	DELPHI [9]	L3 [10–12]		
photon						
x_E range	0.003-1.000	0.018-0.450				
N_γ in range	16.84 ± 0.86	7.37 ± 0.24				
N_γ all x_E	20.97 ± 1.15				20.76	22.65
π^0						
x_E range	0.007-0.400	0.025-1.000	0.011-0.750	0.004-0.150		
N_{π^0} in range	8.29 ± 0.63	4.80 ± 0.32	7.1 ± 0.8	8.38 ± 0.67		
N_{π^0} all x_E	9.55 ± 0.76	9.63 ± 0.64	9.2 ± 1.0	9.18 ± 0.73	9.60	10.29
η						
x_E range	0.025-1.000	0.100-1.000		0.020-0.300		
N_η in range	0.79 ± 0.08	0.282 ± 0.022		0.70 ± 0.08		
N_η all x_E	0.97 ± 0.11			0.91 ± 0.11	1.00	0.92
N_η $x_p > 0.1$	0.344 ± 0.030	0.282 ± 0.022			0.286	0.243

Consistent with JETSET tune where 92% of photons come from π^0's.

Some fraction is non-prompt, from K^0_S, Λ decay

9.6 π^0 per event at Z pole